Queueing and Traffic Flow (2)

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Fundamental Principles of Traffic Flow

- We aim to develop mathematical relationships between flow, density, and speed. This helps in planning, design, and operations of roadway facilities (# lanes, length of turnbays, ramp metering policies, etc.).

- Traffic flow elements such as time-space diagram tell us where on the roadway is the vehicle.
Definition: Flow

- Flow = the rate at which vehicles pass a fixed point (vehicles per hour)

\[ q = N \left( \frac{3600}{t_{measured}} \right) \]

- Density (Concentration) = number of vehicles (N) over a stretch of roadway (L) (vehicles per kilometer)

\[ k = \frac{N}{L} \]

Note: We use k because the word is Konzentration in German

N = number of vehicles passing a point in the roadway in \( t_{measured} \) sec
q = equivalent hourly flow
L = length of roadway
k = density
**Definition:**

- **Time mean speed** ($v_t$) = arithmetic mean of speeds of vehicles passing a point
  \[
  v_t = \frac{1}{N} \sum_{n=1}^{N} v_n
  \]

- **Space mean speed** ($v_s$) = harmonic mean of speeds passing a point during a period of time. It also equals the average speeds over a length of roadway
  \[
  v_s = \frac{\frac{N}{\sum_{n=1}^{N} \frac{1}{v_n}}}{\frac{N}{\sum_{n=1}^{N} t_n}} = \frac{NL}{\sum_{n=1}^{N} t_n}
  \]

Note: many texts denote $v$ as $u$, which were once the same letter.

**Where**
- $N$ = number of vehicles
- $t_n$ = time it takes $n^{th}$ vehicle to traverse a section of highway
- $v_n$ = speed of $n^{th}$ vehicle
- $L$ = length of section of highway
Definition: Headway

- Time headway ($h_t$) = difference between the time when the front of a vehicle arrives at a point on the highway and the time the front of the next vehicle arrives at the same point (in seconds)
- Average Time Headway = Average Travel Time per Unit Distance * Average Space Headway

$$ h_t = t \times h_s $$

- Space headway ($h_s$) = difference in position between the front of a vehicle and the front of the next vehicle (in meters)
- Average Space Headway = Space Mean Speed * Average Time Headway

$$ h_s = v_s \times h_t $$
Problem: qkv

- Four vehicles are traveling at constant speeds between sections X and Y with their positions and speeds observed at an instant in time. An observer at point X observes the four vehicles passing point X during a period of 15 seconds. The speeds of the vehicles are measured as 88, 80, 90, and 72 km/hr respectively. Calculate the flow, density, time mean speed, and space mean speed of the vehicles.
Solution: Flow, Density, Time Mean Speed

- Flow

\[ q = N \left( \frac{3600}{t_{\text{measured}}} \right) = 4 \left( \frac{3600}{15} \right) = 960 \text{veh/hr} \]

- Density

\[ k = \frac{N}{L} = \frac{4}{28} = 142 \text{veh/km} \]

- Time Mean Speed

\[ \bar{v}_t = \frac{1}{N} \sum_{n=1}^{N} v_n = \frac{1}{4} (72 + 90 + 80 + 88) = 82.5 \text{km/hr} \]
Solution: Space Mean Speed

\[
\bar{v}_s = \frac{N}{\sum_{n=1}^{N} \frac{1}{v_i}} = \frac{4}{\frac{1}{72} + \frac{1}{90} + \frac{1}{80} + \frac{1}{88}} = 81.86
\]

\[
t_i = \frac{L}{v_i}
\]

\[
t_A = \frac{L}{v_A} = \frac{0.028}{88} = 0.000318 \text{hr}
\]

\[
t_B = \frac{L}{v_B} = \frac{0.028}{80} = 0.000350 \text{hr}
\]

\[
t_C = \frac{L}{v_C} = \frac{0.028}{90} = 0.000311 \text{hr}
\]

\[
t_D = \frac{L}{v_D} = \frac{0.028}{72} = 0.000389 \text{hr}
\]

\[
\bar{n}_x = \frac{Nl}{\sum_{n=1}^{N} t_n} = \frac{4 \times 0.028}{(0.000318 + 0.000350 + 0.000311 + 0.000389)} = 81.87 \text{km/ hr}
\]
Flow-Density Relationships

- Flow = Density * Space Mean Speed
  \[ q = k v_s \]
- Density = 1 / Space Headway
  \[ k = \frac{1}{h_s} \]
- Space Mean Speed = Flow * Space Headway
  \[ v_s = q h_s \]
- Density = Flow * Time Per Unit Distance
  \[ k = \bar{q} t \]
Fundamental Diagram (traditional)

- When density on the highway is zero, the flow is also zero because there are no vehicles on the highway.
- As density increases, flow increases.
- When the density reaches a maximum jam density \( (k_j) \), flow must be zero because vehicles will line up end to end.
- Flow will also increase to a maximum value \( (q_m) \), increases in density beyond that point result in reductions of flow.
- Where speed is space mean speed (at density = 0, speed is freeflow = \( v_f \)). The upper half of the flow curve is uncongested, the lower half is congested.
- The slope of the flow density curve gives speed.
- Rise/Run = Flow/Density = Vehicles per hour/ Vehicles per km = km / hour.

FIGURE 2: Flow-density, and speed-concentration curves (assuming single-regime, linear speed-concentration model)
Flow vs Density
(I-94, 49th/53rd Avenue, Nov 1 2000)

- Observation differs from model.
- As density rises, speed is unchanged to a point (capacity), and then begins to drop.
- The relationship between flow and density is thus more triangular than parabolic.
Models describing traffic flow can be classed into two categories, microscopic and macroscopic.

- Macroscopic models study traffic as if it were a stream. The most widely used model is the Greenshields model which posited that the relationships between speed and density is linear.

- Microscopic models predict the following behavior of cars (their change in speed and position) as a function of the behavior of the leading vehicle.
Traffic Phase

Figure 4: Traffic phase diagram
Queueing Input-Output Diagram with Phases

Figure 5: Queuing input-output diagram – relationships with phases

Note: Phase 3 may be either Phase 3a or Phase 3d depending on the bottleneck being an ‘active’ bottleneck or the result of a ‘downstream’ bottleneck.
Questions

- Questions?
Key Terms

- Time-space diagram
- Flow, speed, density
- Headway (space and time)
- Space mean speed, time mean speed
- Microscopic, Macroscopic
Variables

- \( d_n \) = distance of \( n \)th vehicle
- \( t_n \) = travel time of \( n \)th vehicle
- \( v_n \) = speed (velocity) of \( n \)th vehicle
- \( h_{t,nm} \) = time headway between vehicles \( n \) and \( m \)
- \( h_{s,nm} \) = space (distance) headway between vehicles \( n \) and \( m \)
- \( q \) = flow past a fixed point (vehicles per hour)
- \( N \) = number of vehicles
- \( t_{measured} \) = time over which measurement takes place (number of seconds)
- \( t \) = travel time
- \( k \) = density (vehicles per km)
- \( L \) = length of roadway section (km)
- \( v_t \) = time mean speed
- \( v_s \) = space mean speed
- \( v_f \) = freeflow (uncongested speed)
- \( k_j \) = jam density
- \( q_m \) = maximum flow
Supplementary Reading

- Revised Monograph on Traffic Flow Theory
  http://www.tfhrc.gov/its/tft/tft.htm