Shockwaves: Birdseye Perspective
Driver’s Perspective
Helicopter Perspective
Shockwaves in Traffic Streams

- Shockwaves can be seen by the cascading of brake lights upstream along a highway.
- They are often caused by a change in capacity on the roadways (4 lanes drops to 3) or an incident, or a traffic signal on an arterial, or a merge on freeway.
- As the capacity (maximum flow) drops from $C_1$ to $C_2$, optimum density also changes.
- Speeds of the vehicles passing the bottleneck will of course be reduced, but the drop in speed will cascade upstream as following vehicles also have to decelerate.
Density Rises but Speed Drops

- The figures illustrate the issues. On the main road, far upstream of the bottleneck, traffic moves at density $k_1$, below capacity ($k_{\text{opt}}$). At the bottleneck, density increases to accommodate the most of the flow, but speed drops.
If the flow rates in the two sections are \( q_1 \) and \( q_2 \), then
\[
q_1 = k_1 v_1 \quad \text{and} \quad q_2 = k_2 v_2
\]

\[
q_2 - q_1 = v_w (k_2 - k_1)
\]

or
\[
v_w = \frac{q_2 - q_1}{k_2 - k_1}
\]

which is the slope of the shockwave line in the figure.

- With \( v_1 \) equal to the space mean speed of vehicles in area 1, the speed relative to the line \( w \) is:

\[
v_{r1} = v_1 - v_w
\]

- The speed of vehicles in area 2 relative to the line \( w \) is

\[
v_{r2} = v_2 - v_w
\]
• The number of vehicles crossing line 2 from area 1 during time period $t$ is

$$N_1 = v_{r_1} k_1 t = (v_1 - v_w) k_1 t$$

similarly:

$$N_2 = v_{r_2} k_2 t = (v_2 - v_w) k_2 t$$

Since the number $N_1 = N_2$, we have

$$v_2 k_2 - v_1 k_1 = v_w (k_2 - k_1)$$
Example 1: Incident

• The traffic flow on a highway is $q_1 = 2000$ veh/hr with speed of $v_1 = 80$ km/hr. As the result of an accident, the road is blocked. The density in the queue is $k_2 = 275$ veh/km. (Jam density, vehicle length = 3.63 meters).

• (A) What is the wave speed ($v_W$)?
• (B) What is the rate at which the queue grows, in units of vehicles per hour ($q$)?
Solution (A)

- (A) At what rate does the queue increase?
- 1. Identify Unknowns:
  
  \[
  k_1 = \frac{q}{v_1} = \frac{2000}{80} = 25 \text{ veh/km}
  \]
  \[
  v_2 = 0, \; q_2 = k_2 v_2 = 0
  \]

- 2. solve for wave speed \((v_w)\)
  
  \[
  v_w = \frac{q_2 - q_1}{k_2 - k_1} = \frac{0 - 2000}{275 - 25} = -8 \text{ km/hr}
  \]

- Conclusion: the queue grows against traffic
Solution (B)

• What is the rate at which the queue grows, in units of vehicles per hour?

\[ N_1 = (v_1 - v_w) k_1 t = (v_2 - v_w) k_2 t = N_2 \]

dropping \[ t \quad (let \ t = 1) \]

\[ v_1 k_1 - v_w k_1 = v_2 k_2 - v_w k_2 \]

\[ q_1 - v_w k_1 = q_2 - v_w k_2 \]

\[ 2000 - (-8) \times 25 = 0 - (-8) \times 275 \]

\[ 2200 \text{veh/hr} = 2200 \text{veh/hr} \]
Problem: Rolling Roadblock

- Flow on a road is $q_1=1800 \text{ veh/hr/lane}$, and the density of $14.4 \text{ veh/km/lane}$. To reduce speeding on a section of highway, a police cruiser decides to implement a rolling roadblock, and to travel in the left lane at the speed limit ($v_2=88 \text{ km/hr}$) for 10 km. No one dares pass. After the police cruiser joins, the platoon density increases to $20 \text{ veh/km/lane}$ and flow drops. How many vehicles (per lane) will be in the platoon when the police car leaves the highway?
0. Solve for Unknowns

Original speed

\[ v_1 = \frac{q_1}{k_1} = \frac{1800}{14.4} = 125 \text{ km/hr} \]

Flow after police cruiser joins

\[ q_2 = k_2v_2 = 88 \times 20 = 1760 \text{ veh/hr} \]

1. Calculate the wave velocity:

\[ v_w = \frac{q_2 - q_1}{k_2 - k_1} = \frac{1760 - 1800}{20 - 14.4} = -7.14 \text{ km/hr} \]

2. Determine the growth rate of the platoon (relative speed)

\[ v_{r2} = v_2 - v_w = 88 - (-7.14) = 95.1 \text{ km/hr} \]
3. Determine the time spent by the police cruiser on the highway
10 km / 88 km/hr = 0.11 hr = 6.8 minutes

4. Calculate the Length of platoon (not a standing queue)
95.1 km/hr * 0.11 hr = 10.46 km

5. What is the rate at which the queue grows, in units of vehicles per hour?
\[ q_1 - k_1 v_w = q_2 - k_2 v_w = 1800 - (14.4 \times 7.14) = 1760 - (20 \times 7.14) = 1902 \text{ veh/hr} \]

6. The number of vehicles in platoon
\[ L_p k_2 = 10.46 \text{ km} \times 20 \text{ veh/km} = 209.2 \text{ vehicles OR} \]
\[ = 1902 \text{ veh/hr} \times 0.11 \text{ hr} = 209.2 \text{ veh} \]
QUESTIONS?
Key Terms

- Shockwaves
- Time lag, space lag
Variables

- \( q \) = flow
- \( c \) = capacity (maximum flow)
- \( k \) = density
- \( v \) = speed
- \( v_r \) = relative speed (travel speed minus wave speed)
- \( v_w \) = wave speed
- \( N \) = number of vehicles crossing wave boundary