Traffic Analysis at Signalized Intersections 2

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Analysis of Signalized Intersections

D/D/1 Queueing
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Approach arrival rate is less than approach capacity.

Results in Fig. where:
- \( \lambda \) = the arrival rate (veh/sec)
- \( \mu \) = the departure rate (veh/sec)
- \( g \) = effective green (sec)
- \( r \) = effective red (sec)
- \( t \) = total transpired time (sec)

“Arrivals \( \lambda t \)” line gives the total no. of vehicle arrivals at time “\( t \)”. “Departures \( \mu t \)” line gives the slope of the departures (no. of vehicles that depart) during effective green (\( g \)).

\( (t_c) \) time(sec) from the start of effective green until the queue dissipates.

\( (C) \) cycle length

Note:
1. Per cycle approach arrivals = \( \lambda C \)
2. Corresponding approach capacities (departures) per cycle = \( \mu g \)

Therefore: \( \mu g \) exceeds \( \lambda C \) for all cycles (no spillback)
Time to Queue dissipation, $t_c$, is

\[ t_c = \frac{\rho r}{1-\rho} \]

recall: $\rho = \frac{\lambda}{\mu}$
Proportion of cycle with queue, $P_q$, is

\[ P_q = \frac{r + t_c}{C} \]
Proportion of vehicles stopped, $P_s$, is:

$$P_s = \frac{\lambda (r + t_C)}{\lambda (r + g)} = \frac{r + t_C}{\lambda C} = \rho C$$

$$P_s = \frac{\lambda (r + t_C)}{\lambda (r + g)} = \frac{\mu t_C}{\lambda C} = \frac{\mu t_C}{\rho C}$$
Maximum no. of vehicles in queue, $Q_{\text{max}}$, is

$$Q_{\text{max}} = \lambda r$$
Total vehicle delay per cycle, $D_t$, is:

$$D_t = \frac{\lambda r^2}{2(1-\rho)}$$
Average vehicle delay per cycle, $d_{\text{avg}}$, is:

$$d_{\text{avg}} = \frac{\lambda r^2}{2(1-\rho)} \times \frac{1}{\lambda C}$$

$$d_{\text{avg}} = \frac{r^2}{2C(1-\rho)}$$
Maximum delay of any vehicle, \( d_{\text{max}} \) is

\[ d_{\text{max}} = r \]
Example 7.1
Mannering Kilareski Washburn

An approach at a pretimed signalized intersection has a saturation flow of 2400 veh/h and is allocated 24 seconds of effective green in an 80-second signal cycle. If the flow at the approach is 500 veh/h, provide an analysis of the intersection assuming D/D/1 queuing.
Solution 1

- Putting arrival and departure rates into common units of vehicles per second
  - $\lambda = \frac{500}{3600} = 0.139 \text{ veh/s}$
  - $\mu = \frac{2400}{3600} = 0.667 \text{ veh/s}$
  - Intensity $\rho = \frac{\lambda}{\mu} = 0.208$
Solution 2

Ensure that capacity exceeds arrivals, note that the capacity ($\mu g$) is 16 veh/cycle (0.667x24) which is greater than (permitting fractions of vehicles for the sake of clarity) the 11.12 arrivals ($\lambda C=0.139x80$).

By definition $r=C-g = 80 - 24 = 56$ s

This leads to the following values:
# Solution 3

<table>
<thead>
<tr>
<th>Property</th>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to queue clearance after the start of the effective green</td>
<td>$t_c = \frac{\rho r}{1-\rho}$</td>
<td>14.71s</td>
</tr>
<tr>
<td>Proportion of the cycle with a queue</td>
<td>$P_q = \frac{r + t_c}{C}$</td>
<td>0.884</td>
</tr>
<tr>
<td>Proportion of vehicles stopped</td>
<td>$P_s = \frac{\frac{\lambda (r+t_c)}{\lambda (r+g)}}{\frac{\mu t_c}{\lambda C}} = \frac{\mu t_c}{\rho C}$</td>
<td>0.884</td>
</tr>
<tr>
<td>Maximum number of vehicles in the queue</td>
<td>$Q_{\text{max}} = \lambda r$</td>
<td>7.78</td>
</tr>
<tr>
<td>Total vehicle delay per cycle</td>
<td>$D_t = \frac{\lambda r^2}{2(1-p)}$</td>
<td>275.19 veh-s</td>
</tr>
<tr>
<td>Average delay per vehicle</td>
<td>$d_{\text{avg}} = \frac{\lambda r^2}{2(1-p)} * \frac{1}{\lambda C}$</td>
<td>24.75s</td>
</tr>
<tr>
<td>Maximum delay of any vehicle</td>
<td>$d_{\text{max}} = r$</td>
<td>56s</td>
</tr>
</tbody>
</table>
An approach to a pretimed signalized intersection has a saturated flow of 1700 veh/h. The signal’s cycle length is 60 seconds and the approach’s effective red is 40 seconds. During three consecutive cycles, 15, 8 and 4 vehicles arrive. Determine the total vehicle delay over the three cycles assuming D/D/1 queueing.
Solution 1

For all cycles, the departure rate is $\mu = 1700 \text{ veh/h} / 3600 \text{ s/h} = 0.472 \text{ veh/s}$

During the first cycle, the number of vehicles that will depart from the signal is (permitting fractions for the sake of clarity) $\mu g = 0.472 (20) = 9.44 \text{ veh}$.

Therefore $15 - 9.44 = 5.56$ vehicles will not be able to pass through the intersection the first cycle even though they arrived during the first cycle.
Solution 2

At the end of the second cycle, 23 vehicle (15 +8) will have arrived, but only 18.88 (2µg) will have departed, leaving 4.12 vehicles waiting at the beginning of the third cycle. At the end of the third cycle, a total of 27 vehicles will have arrived and as many as 28.32 (3µg) could have departed, so the queue that began to form during the first cycle will dissipate at some time during the third cycle, as seen in the figure.
Solution 3

Note computing $D_3$ requires knowing when in the cycle the queue dissipates. The time to queue clearance after the start of the effective green $t_c$ is (with $\lambda_3$ being the arrival rate during the third cycle ($4$ veh/60 sec = $0.067$ veh/s) and $n_3$ being the number of vehicles in the queue at the start of the third cycle)

$$n_3 + \lambda_3(r + t_c) = \mu t_c$$

So: $(23-18.88) + 0.067(40+t_c)=0.472t_c$

which gives $t_c = 16.8$ seconds. The queue clears at 56.8 seconds = $40 + 16.8$ seconds, when 26.8 vehicles will have arrived.
### Solution 4

<table>
<thead>
<tr>
<th></th>
<th>Formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>D₁</td>
<td>( \frac{1}{2}(60)(15) - \frac{1}{2}(20)(9.44) )</td>
<td>355.6 veh-s</td>
</tr>
<tr>
<td>D₂</td>
<td>( \frac{1}{2}(60)(15+23) - (40)(9.44) - \frac{1}{2}(20)(9.44 + 18.88) )</td>
<td>479.2 veh-s</td>
</tr>
<tr>
<td>D₃</td>
<td>( \frac{1}{2}(56.8)(23+26.8) - (40)(18.88) - \frac{1}{2}(16.8)(18.88 + 26.8) )</td>
<td>275.4 veh-s</td>
</tr>
</tbody>
</table>

\[ D_t = D_1 + D_2 + D_3 = 1110.2 \text{ veh-s} \]

**Figure 6.2** \( D/D/1 \) queuing diagram for Example 6.2.
Optimal Traffic Signal Timing

- Allocating effective green times to competing approaches in an optimal fashion

- A set of factors involved:
  1. what are the arrival pattern
  2. On what basis should the signal timing be optimized? Delay, # of stops, or queue length, etc?
  3. Other factors, such as coordination
Minimize vehicle delay

D/D/1 queue

A pretimed signal controls a four-way intersection with no turning permitted and zero lost time. The EB and WB traffic volumes are 700 and 800 veh/h, and the two movements share the same effective green and effective red portions of cycles. The NB and SB directions also share cycle times, with volumes of 400 and 250 veh/h, respectively. If the saturation flow of all approaches is 1800 veh/h, the cycle length is 60 seconds, and D/D/1 queuing applies, determine the effective red and green times that must be allocated to each directions combination (N-S, E-W) to minimize the total vehicle delay and compute the total delay per cycle.
Solution 1

Put arrival and departure rates into common units of vehicles per second, compute intensity:

<table>
<thead>
<tr>
<th>Direction</th>
<th>Rate (λ)</th>
<th>Intensity $\rho=\frac{\lambda}{\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EB</td>
<td>700 veh/h / 3600 s/h = 0.194 veh/s</td>
<td>0.388</td>
</tr>
<tr>
<td>WB</td>
<td>800 veh/h / 3600 s/h = 0.222 veh/s</td>
<td>0.444</td>
</tr>
<tr>
<td>NB</td>
<td>400 veh/h / 3600 s/h = 0.111 veh/s</td>
<td>0.222</td>
</tr>
<tr>
<td>SB</td>
<td>50 veh/h / 3600 s/h = 0.069 veh/s</td>
<td>0.138</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1800 veh/h / 3600 s/h = 0.5 veh/s</td>
<td></td>
</tr>
</tbody>
</table>
Since the departure rate is the same for all approaches, the traffic intensities are easily computed. If it is assumed that approach capacity exceeds approach arrivals for all approaches, the total delay at the intersection is:

\[
D_t = \frac{\lambda_{EB}r_{EB}^2}{2(1-\rho_{EB})} + \frac{\lambda_{WB}r_{WB}^2}{2(1-\rho_{WB})} + \frac{\lambda_{NB}r_{NB}^2}{2(1-\rho_{NB})} + \frac{\lambda_{SB}r_{SB}^2}{2(1-\rho_{SB})}
\]

Substituting

\[
D_t = 0.1585r_{EB}^2 + 0.1996r_{WB}^2 + 0.7115r_{NB}^2 + 0.04r_{SB}^2
\]
Solution 3

The problem states that the east and west effective reds are equal and north and south effective reds are equal. So let $r_{EW}$ be the effective red of east and westbound directions ($r_{EW}=r_{EB}=r_{WB}$) and let $r_{NS}$ be the effective red of northbound and southbound directions ($r_{NS}=r_{NB}=r_{SB}$). By definition, with a 60-second signal cycle, $r_{NS}=60-r_{EW}$. Substituting this into the total delay expression gives:

$$D_t = 0.1585 r_{EW}^2 + 0.1996 r_{EW}^2 + 0.7115 (60 - r_{EW})^2 + 0.04 (60 - r_{EW})^2$$

$$= 0.46925 r_{EW}^2 - 13.338 r_{EW} + 400.14$$
At minimum total delay, we set the first derivative equal to zero

\[
\frac{\partial D_t}{\partial r_{EW}} = 0.9357 r_{EW} - 13.338 = 0
\]

which gives \( r_{EW} = 14.2 \text{s} \) (\( g_{EW} = 45.8 \text{ s} \)) and \( r_{NS} = 45.8 \text{s} \) (\( g_{NS} = 14.2 \text{s} \)). Total delay can be found by substituting, \( D_t = 305.36 \text{ veh-sec} \)
Finally a check of the earlier assumption that the approach capacity exceeds approach arrivals for all approaches must be undertaken. In the 60-second cycle, eastbound and westbound vehicle arrivals are 11.64 (0.194 x 60) and 13.32 (0.222 x 60) respectively. With 45.8 seconds of effective green, the capacity for both approaches is 22.9 (0.5 x 45.8) vehicles, so the assumption is satisfied. The northbound and southbound vehicle arrivals are 6.67 (0.111 x 60) and 4.14 (0.069 x 60) respectively and with 14.2 seconds of effective green, the capacity for both approaches is 7.1 (0.5 x 14.2) vehicles. Again the assumption is satisfied and the method used in this example is valid.
Questions

- Questions?
Abbreviations
Key Terms
Variables