Stopping Distance

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For highway design it is often useful to have a braking equation where the deceleration is caused by a friction force generated by skidding tires, and where the vehicle might be braking on a down (or up) grade. To do this assume that the vehicle is a simple object with mass $m$ sliding on an inclined surface, where the angle of incline equals $\theta$.

Two forces are operating on the object, a gravitational force tending to pull it downhill, and a friction force tending to resist the downhill motion. Taking positive forces to be pulling in the downhill direction, the total force on the object can be written as

$$ F = W \left( \sin(\theta) - f \cos(\theta) \right) $$

where $W = mg =$ object’s weight,
$f =$ coefficient of friction.
Using Newton’s second law we can conclude then that the acceleration (a) of the object is

\[ a = g \left( \sin(\theta) - f \cos(\theta) \right) \]
Equations for Braking Distance

Using our basic equations to solve for braking distance \((d_b)\) in terms of initial speed \((v_i)\) and ending speed \((v_e)\) gives

\[
d_b = \frac{v_i^2 - v_e^2}{-2a}
\]
Substituting and substituting for the acceleration yields

\[ d_b = \frac{v_i^2 - v_e^2}{2g(f \cos(\theta) - \sin(\theta))} \]
For angles commonly encountered on roads, $\cos(\theta) \approx 1$ and $\sin(\theta) \approx \tan(\theta) = G$, where $G$ is called the road’s grade. This gives

$$d_b = \frac{v_i^2 - v_e^2}{2g(f - G)}$$
Example 1: Stopping distance

Example: A vehicle initially traveling at 88 km/h skids to a stop on a 3% downgrade, where the pavement surface provides a coefficient of friction equal to 0.3. How far does the vehicle travel before coming to a stop?
Solution

\[ d_b = \frac{\left( 88 \times \left( \frac{1000}{3600} \right) \right)^2 - (0)^2}{2 \times (9.8) \times (0.3 - 0.03)} = 112.9 \text{ m} \]
Example 2: Braking Point

A motorist traveling at 88 km/h on a freeway intends to leave the roadway using an exit ramp with a maximum allowable speed of 50 km/h. At what point should the driver step on her brakes to reduce her speed to the ramp's speed limit as she enters the ramp? Assume the coefficient of friction is 0.3 and the alignment of the road is horizontal.
Solution

\[ d_b = \frac{\left( 88 \times \left( \frac{1000}{3600} \right) \right)^2 - \left( 50 \times \left( \frac{1000}{3600} \right) \right)^2}{2 \times (9.8) \times (0.3 - 0)} = 68.8m \]
Example: Coefficient of Friction

A vehicle initially traveling at 150 km/hr skids to a stop on a 3% downgrade, taking 200 m to do so. What is the coefficient of friction on this surface?
Solution

\[ d_b = \left( \frac{150 \times \left( \frac{1000}{3600} \right)}{2 \times (9.8) \times (f - 0.03)} \right)^2 - (0)^2 = 200m \]

\[ (f - 0.03) = \left( \frac{150 \times \left( \frac{1000}{3600} \right)}{2 \times (9.8) \times 200} \right)^2 - (0)^2 \]

\[ f = 0.47 \]
Example 3: Grade

What should the grade be for the previous example if the coefficient of friction is 0.40?
Thus the road needs to be uphill if the trucks are going that speed on that surface and can stop that quickly.

\[ d_b = \frac{\left( 150 \times \left( \frac{1000}{3600} \right) \right)^2 - (0)^2}{2 \times (9.8) \times (0.40 - G)} = 200m \]

\[ (0.40 - G) = \frac{\left( 150 \times \left( \frac{1000}{3600} \right) \right)^2 - (0)^2}{2 \times (9.8) \times 200} \]

\[ G = 0.40 - 0.44 = -0.04 \]
Question

If the coefficient of friction is 0 (zero) and the grade is 0, how long does it take a moving vehicle to stop?
[Note, the design conditions for roads are wet, i.e. a lower coefficient of friction]
Problem 1:

You see a drunk University of Wisconsin student lying across the road, and, to avoid the paperwork, decide to stop.

If your vehicle was initially traveling at 100 km/h and skids to a stop on a 2.5% upgrade, taking 75 m to do so, what was the coefficient of friction on this surface?
Solution

\[ d_b = \frac{\left(100 \cdot \left(\frac{1000}{3600}\right)\right)^2 - (0)^2}{2 \cdot (9.8) \cdot (f + 0.025)} = 75 \text{m} \]

\[ (f + 0.025) = \frac{(27.78)^2}{2 \cdot (9.8) \cdot 75} \]

\[ f = 0.50 \]
**Accident Reconstruction**

You want to estimate the speed of a vehicle just before it was involved in an accident. You have measured skid marks and an estimate of the speed at impact. Your unknowns are the initial speed and the coefficient of friction.

- First, average the lengths of skid marks
- Second, estimate the coefficient of friction by doing a trial run
- Third, estimate the unknown velocity
Example 4: Accident Reconstruction

You are shown an accident scene with a vehicle and a light pole. The vehicle was estimated to hit the light pole at 50 km/hr.

The skid marks are measured to be 210, 205, 190, and 195 meters.
Solution

First, Average the Skid Marks.

\[
\frac{(210+205+190+195)}{4} = 200
\]
Second, a trial run is performed to estimate the coefficient of friction.

A car traveling 60 km/hr can stop in 100 meters.

\[
d_b = \frac{\left(60 \times \left(\frac{1000}{3600}\right)\right)^2 - (0)^2}{2 \times (9.8) \times (f - 0)} = 100m
\]

\[
f = \frac{\left(60 \times \left(\frac{1000}{3600}\right)\right)^2 - (0)^2}{2 \times (9.8) \times 100}
\]

\[f = 0.14\]
Solution

Third, estimate the unknown velocity

\[
d_b = \frac{\left( v \left( \frac{1000}{3600} \right) \right)^2 - \left( 50 \left( \frac{1000}{3600} \right) \right)^2}{2 \times 9.8 \times (0.14 - 0)} = 200m
\]

\[
\left( v \left( \frac{1000}{3600} \right) \right)^2 - \left( 50 \left( \frac{1000}{3600} \right) \right)^2 = 200m \times (2 \times 9.8 \times (0.14))
\]

\[
548.8 + 192.9 = v^2 \left( \frac{1000}{3600} \right)^2
\]

\[
v^2 = \frac{741.7}{0.077} = 9612.43
\]

\[
v = 98km / h
\]
Finally, for highway design it is often useful to have a stopping distance formula that includes the distance traveled during the driver's perception/reaction time \( t \) and which allows speeds to be expressed directly in units of km/h. Making these changes gives us the AASHTO stopping sight distance formula

\[
d_s = d_r + d_b = 0.278 t_r v_i + \frac{v_i^2}{254(f \pm G)}
\]

where

- \( d_s \) = stopping (sight) distance (m)
- \( d_r \) = perception reaction distance (m)
- \( d_b \) = braking distance (m)
- \( v_i \) = initial speed (km/h)
- \( t_r \) = perception/reaction time (seconds)
- \( f \) = AASHTO stopping friction coefficient (dimensionless) (see Table 3.3)
- \( G \) = roadway grade (dimensionless)
Example 4: SSD

Calculate the Stopping Sight Distance from the previous example

Stopping Sight Distance = \((\frac{1000}{3600}) \times 98 \times 2.5 + \frac{98^2}{(254 \times 0.14)}\) = 338

Where Perception Reaction Time = 2.5, Braking Distance = 200 m

(but not at 0 km/h, at point of impact)

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<th>Design Speed (km/h)</th>
<th>Coefficient of Skidding Friction (f)</th>
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<tr>
<td>120</td>
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Questions

Questions?
Key Terms

- stopping distance
- braking distance
- reaction distance
Variables

- $d_s$ = stopping (sight) distance (m)
- $d_r$ = perception reaction distance (m)
- $d_b$ = braking distance (m)
- $v_i, v_e$ = initial, ending speed (km/hr)
- $t_r$ = perception/reaction time (seconds)
- $f$ = AASHTO stopping friction coefficient (dimensionless) (see Table 3.3)
- $G$ = roadway grade (dimensionless)
- $W = mg$ = object's weight,
- $f$ = coefficient of friction.