Vertical Curves

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Vertical Curves

- Vertical curves provide a means to smoothly shift from one tangent grade to another. They are usually parabolic in shape. They are classified into crest vertical curves and sag vertical curves.

- In highway design, most vertical curves are equal-tangent curves, which means that the horizontal distance from the center of the curve to the end of the curve is identical in both directions.

- (Unequal-tangent vertical curves, which are simply equal-tangent curves that have been attached to one another, are seldom used.)

- In highway design, the grades of the disjointed segments of roadway are normally known before any vertical curve calculations are initiated.

- In addition, the design speed of the roadway, the stopping sight distance, and the decision sight distance are also well established.

- The first step in the design of a vertical curve is the calculation of the curve length, which is the length of the curve as it would appear when projected on the x-axis.
Stopping Sight Distance and Vertical Curvature

- Because the stopping sight distance (S or dₜ) should always be adequate, the length of the curve depends on the stopping sight distance.

- Occasionally, as with any other section of a highway, the decision sight distance is a more appropriate sight distance. In these instances, the decision sight distance governs the length of the vertical curve.

- The curve length calculations are slightly different for sag and crest vertical curves, and are covered separately.

- Let’s assume that you have already calculated the appropriate length (L) for your curve.

- The first step in developing the profile for your curve is to find the center of your curve. The location of the center-point is where the disjointed segments of the roadway would have intersected, had they been allowed to do so. In other words, draw lines tangent to your roadway segments and see where those lines intersect. This intersection is normally called the vertical point of intersection (PVI).
PVC and PVT

- The vertical point of curvature (PVC) and the vertical point of tangency (PVT) are located a horizontal distance of L/2 from the PVI. The PVC is generally designated as the origin for the curve and is located on the approaching roadway segment. The PVT serves as the end of the vertical curve and is located at the point where the vertical curve connects with the departing roadway segment. In other words, the PVC and PVT are the points along the roadway where the vertical curve begins and ends.

- One you have located the PVI, PVC, and PVT, you are ready to develop the shape of your curve. The equation that calculates the elevation at every point along an equal-tangent parabolic vertical curve is shown below.
Equation of Elevation

\[ y = ax^2 + bx + c \]

Where:

\( y \) = Elevation of the curve at a distance \( x \) meters from the PVC (m)

\( c \) = elevation of PVC (m)

\( b = G_1 \)

\( a = \frac{(G_2 - G_1)}{2L} \)

\( L \) = Length of the curve (m)

\( x \) = Horizontal distance from the PVC (m) (Varied from 0 to L for graphing.)
Example 1

A 150 m equal tangent crest-vertical curve has the PVC at station 8+500 and elevation of 250 m.

The initial grade is +2.6% and the final grade is -0.5%.

Determine the elevation and stationing of the PVI, PVT and the highest point of the curve.
Solution 1

Because the curve is equal tangent, the PVI will be 75 m from the PVC, and the PVT will be 150 m from the PVC.

The stationing is therefore at:
- PVI: 8+575 &
- PVT: 8+650

The grade = +2.6% = 0.026 m/m

PVI elevation is thus 250 + 0.026*75 = 251.95 m

PVT elevation is 251.95 - 0.005*75 = 251.575 m

Maximum occurs when first derivative = 0

\[
\frac{dy}{dx} = 2ax+b = 0
\]

b = \( G_1 \) = 0.026

\[
a = \frac{(G_2-G_1)}{2L} = \frac{(-0.005-0.026)}{(2*150)} = -0.0001033
\]

x = 125.8 m

- Substituting

0 = 2(-0.0001)x + 0.026

Stationing of max = 8+500 + 0+125.8 = 8+625.8

- Elevation = \( y = ax^2 + bx + c \)

= (-0.0001)*125.8^2 + 0.026*125.8 + 250 = -1.583 + 3.27 + 250 = 251.69

- 251.69 < 251.95 (PVI) check
Suppose $G_1=0$

- Maximum occurs when first derivative = 0
- $\frac{dy}{dx} = 2ax + b = 0$
- $b = G_1 = 0$
- $a = \frac{(G_2 - G_1)}{2L} = \frac{-0.005}{2 * 150} = -0.0000167$
- $x = 0\text{ m}$

So maximum is at start of curve ($x=0$)

Suppose $G_2 = 0$

- $b = G_1 = 0.026$
- $a = \frac{(G_2 - G_1)}{2L} = \frac{-0.026}{300} = 0.0000867$
- $\frac{dy}{dx} = 2ax + b = 0 = -0.000173x + 0.026 = 0$
- $x = 150\text{ m}$

So maximum is at end of curve ($x=150$)
Crest vertical curves connect inclined sections of roadway, forming a crest. Objective: find an appropriate length for the curve that will accommodate the correct stopping sight distance.

Crest Vertical Curves: Design

Crest vertical curves connect inclined sections of roadway, forming a crest. Objective: find an appropriate length for the curve that will accommodate the correct stopping sight distance.
Equations Relating Length of Curve and Sight Distance

If \( S \leq L \) then
\[
L = \frac{200 \left( \sqrt{h_1} + \sqrt{h_2} \right)^2}{A}
\]

If \( S \geq L \) then
\[
L = 2S - \frac{200 \left( \sqrt{h_1} + \sqrt{h_2} \right)^2}{A}
\]

• Where:
  - \( L \) = Length of the crest vertical curve (m)
  - \( S \) = Sight distance (m)
  - \( A = |G_2-G_1| \) = The absolute value of change in grades (as a percent)
  - \( h_1 \) = Height of the driver's eyes above the ground (m)
  - \( h_2 \) = Height of the object above the roadway (m)
Comments

• The heights in the calculations above should be those that correspond to the sight distance of interest.
  • For the stopping sight distance, $h_1 = 1.1 \text{ m}$ and $h_2 = 0.15 \text{ m}$.
  • For the passing sight distance, $h_1 = 1.1 \text{ m}$ and $h_2 = 1.3 \text{ m}$.

• While the sight distance has been portrayed as the only parameter that affects the design of vertical curves, this isn't entirely true. Vertical curves should also be comfortable for the driver, aesthetically pleasing, safe, and capable of facilitating proper drainage. In the special case of crest vertical curves, it just so happens that a curve designed with adequate sight distances in mind is usually aesthetically pleasing and comfortable for the driver. In addition, drainage is rarely a special concern.
Example 2: Minimum Length of Crest Vertical Curve

A crest vertical curve is to be designed to join a +3 percent grade with a -3 percent grade at a section of a two lane highway.

Determine the minimum length of the curve if the design speed (V) of the highway is 100 km/hr and S < L.

Assume that \( f = 0.29 \) and PRT = 2.5 sec
Solution

Determine SSD  (Since the grade is constantly changing, the worst case value for G is used).

\[ S = d_s = 0.278t_rv + \frac{v^2}{254(f \pm G)} \]

\[ = 0.278 \times 2.5 \times 100 + \frac{100^2}{254(0.29 - 0.03)} = 221m \]

Obtain minimum length of the vertical curve

\[ L = \frac{AS^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2} = \frac{6 \times 221^2}{200\left(\sqrt{1.1} + \sqrt{1.5}\right)^2} = 710m \]
Problem: Minimum Safe Speed on a Crest Vertical Curve

- An existing vertical curve on a highway joins a +4 percent grade with a -4 percent grade.
- If the length of the curve is 100 m, what is the maximum safe speed on the curve.
- Assume $f=0.4$ and $PRT = 2.5$ sec. Also $S < L$
Solution

First determine the safe stopping distance $S$ (SSD) using the length of the curve

$$L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} = 100 = \frac{8S^2}{200(\sqrt{1.1} + \sqrt{1.5})^2}$$

$$S = 71.7m$$

Now determine the maximum safe speed for this SSD

$$S = ds = 0.278tv + \frac{v^2}{254(f \pm G)} = 71.7 = 0.278*2.5v + \frac{v^2}{254(0.4 - 0.04)}$$

$$= 0.69v + 0.010936v^2$$

$$71.7 = 0.69v + 0.010936v^2$$

$$v^2 + 63.6v - 6556 = 0$$

$$v \approx 55km / h$$

(Technically 55.2, but round down to nearest speed limit in 5 km/hr increments)
Problem: Design of a Crest Vertical Curve

A crest vertical curve joining a +4 percent and -3 percent grade is designed for 120 km/hr. If the tangents intersect at metric station 20 + 050.00 and at an elevation of 150 meters, determine the stations and elevations for the PVI and PVT. Also calculate the elevations of intermediate points on the curve at the whole stations.

From tables, f=0.28
Solution

Stopping Sight distance (G=0.04 is critical grade)

\[ S = \frac{d_s}{2} = 0.278t_v + \frac{v^2}{254(f \pm G)} = 0.278(2.5)120 + \frac{120^2}{254(0.28 - 0.04)} = 320 m \]

• Length of Curve

\[ L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{7 \times 320^2}{200(\sqrt{1.1} + \sqrt{0.15})^2} = 1737.78 m \]

• Station of PVC = (20+050.00) - (1+737.78)/2 = (20+050.00) - (0+868.89) = 19+181.11
• Station of PVT = (20+050.00) + (1+737.78)/2 = 20+918.89
• Elevation of PVC = PV1y-(G*L/2)=150 - (0.04*0.5*1737.78) = 115.24 m
• Elevation of PVT = PV1y-(G*L/2)=150 - (0.03*0.5*1737.78) = 123.93 m
Questions

Questions?
Abbreviations

- PVI - Point of Vertical Intersection (sometimes VPI)
- PVC - Point of Vertical Curvature (sometimes VPC)
- PVT - Point of Vertical Tangency (sometimes VPT)
- SSD - Stopping sight distance ($d_s$) or (S)
Key Terms

- Sag vertical curve
- Crest vertical curve
y = Elevation of the curve at a distance x meters from the PVC (m)
c = elevation of PVC (m)
b = G₁
a = (G₂ - G₁) / 2L
L = Length of the crest vertical curve (m)
S = Sight distance (m)
A = The change in grades (|G₂ - G₁| as a percent)
h₁ = Height of the driver's eyes above the ground (m)
h₂ = Height of the object above the roadway (m)