Agent-Based Model of Price Competition, Capacity Choice, and Product Differentiation on Congested Networks

Lei Zhang, David M. Levinson, and Shanjiang Zhu

Address for correspondence: Lei Zhang, School of Civil and Construction Engineering, Oregon State University, 220 Owen Hall, Corvallis, OR 97331 (lei.zhang@oregonstate.edu). David M. Levinson is RP Braun/CTS Chair in Transportation Engineering, Department of Civil Engineering, University of Minnesota. Shanjiang Zhu is Graduate Research Assistant, Department of Civil Engineering, University of Minnesota.

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Abstract

Using consistent agent-based techniques, this research explores the welfare consequences of product differentiation on congested networks. The economic analysis focuses on the source, evolution, measurement, and impact of product differentiation with heterogeneous users on a mixed ownership network. Path differentiation and space differentiation are defined and measured for a base scenario and several variants. The findings favour a fixed-rate road pricing policy compared to complete pricing freedom on toll roads. It is also shown that the impact of production differentiation on welfare is not always positive and depends on the level of user heterogeneity.

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1.0 Introduction

In markets with heterogeneous users who have a distinct preference and willingness to pay, firms often achieve their welfare or revenue objectives by supplying a differentiated quality of products or services associated with differentiated prices. This product differentiation is seen in the operations of airlines and passenger railways, where users are allowed to pay a premium to receive better service in a first-class cabin. If we define all variable monetary costs that users pay for transportation services as the price, and travel time as the measure of service level on a road network, product differentiation does not exist under the standard Wardrop’s user equilibrium without tolls, which states that all used routes between an origin–destination pair have the same lowest travel time. Due to welfare and/or revenue considerations, public and private road authorities may impose tolls on users of certain facilities, creating price differences on alternative routes serving the same origin–destination pair. As users adjust their travel decisions in response to the price discrepancy, the network can exhibit lower levels of congestion on tolled routes and higher levels of congestion on untolled routes, giving rise to product differentiation. Over time, capacity choices by facility providers also affect the level of service both directly, by mitigating congestion, and indirectly, by influencing future pricing decisions.

Transportation economists have long promoted welfare-maximising marginal-cost tolls on roads, and various other road-pricing schemes under second-best conditions (for example, in cases when not all roads can be tolled, or when there exist cross-subsidies between modes). Politicians may favour road pricing for its revenue-generating and congestion-mitigating potentials. The emergence of private roads has introduced profit-maximising objectives into road-pricing analysis. Few researchers consider price and capacity choices simultaneously, and none considers those choices on general networks with complex substitutional and complementary effects. Product differentiation and user heterogeneity are recognised as important factors in the welfare analysis of road-pricing policies. However, proper measures of product differentiation on road networks remain largely unavailable to researchers.

This paper models the toll and capacity choices on general transportation networks with complete user heterogeneity, and the subsequent emergence of product differentiation. The overall and distributional effects of product differentiation are analysed for several system scenarios. The modelling methodology in this paper represents a significant departure from the previous literature in transportation economics that has relied primarily on equilibrium analysis on stylised networks. An evolutionary
modelling approach is adopted herein, which tracks the choices by individual decision-makers over time. Users make decisions regarding trip frequencies, destinations, and routes, and continuously adjust these decisions according to their heterogeneous preferences, values of time, and the pricing/capacity decisions by road authorities. Given an initial system state and carefully calibrated parameters, the transportation network in the model evolves into a specific pattern, replicating the central tendency of real-world network changes over time. This agent-based approach is applicable to large real-world transportation networks, and is demonstrated in this paper on a revised Sioux Falls network with both tolled and untolled roads.

2.0 Literature Review

Theoretical studies about road pricing have a long history (Dupuit, 1844; Pigou, 1920; Knight, 1924; Mohring and Harwitz, 1962; Vickery, 1963; Walters, 1968; Small, 1992; Arnott et al., 1993; Button and Verhoef, 1998; Gomez-Ibanez, 1999; Liu and McDonald, 1999; de Palma and Lindsey, 2002; Verhoef, 2002; Levinson, 2005; Zou and Levinson, 2006). Research in this field has advanced from first-best to various second-best pricing schemes, from time-independent to time-varying tolls, from assumptions of user homogeneity to heterogeneity, and from assumptions of a centralised system operator to various ownership regimes. The majority of previous literature on road pricing (second-based congestion pricing, in particular) is based on a small parallel network; few studies examine price competition on a serial network (Levinson, 1999, 2000; De Borger et al., 2006). The use of small stylised networks for transportation economics analysis allows researchers to focus on the welfare consequences of alternative policies. Verhoef (2002) develops an algorithm to identify toll points and second-best tolls on a general network.

While pricing policies are typically proposed with the goal of improving short-run network efficiency, studies on investment principles are generally concerned with long-run efficiency assuming a priori the pricing policy (Wohl and Hendrickson, 1984). Previous research on the network design problem (NDP) seeks to find the optimal network that serves a certain travel demand, or the optimal network enhancement given a budget constraint (Boyce et al., 1974; LeBlanc, 1975; Poorzahedy and Turnquist, 1982; Yang and Bell, 1998; Meng et al., 2001). Game theoretical approaches have also found their way into road pricing and investment analysis, with the clear advantage of explicitly considering individual
players, their strategies, and interactions (Hollander and Prashker, 2006). Benefit–cost analysis has been extensively used in practice for strategic investment planning.

Few studies consider price competition and capacity choices together. Keeler and Small (1977) develop a theoretical model to examine optimal peak-load pricing and investment on urban expressways. Verhoef and Rouwendal (2004) recently revisited this topic with additional considerations of second-best pricing policies. Zhang and Levinson (2006) consider pricing, investment, and ownership dynamics simultaneously. Although the benefits of jointly considering price and capacity competition are obvious as they clearly influence each other, these studies show that the associated modelling efforts also increase significantly.

Assuming away user heterogeneity and the possibility of product differentiation could cause the benefit of road pricing to be underestimated, as shown by several previous studies (Arnott et al., 1992; de Palma, 1992; Schmanske, 1993; Small and Yan, 2001; Verhoef and Small, 2004). It has been shown that with heterogeneity (for example, different values of time), a single toll is inferior to differentiated tolls on two parallel roads. Verhoef and Small (2004) base their analysis on a continuous value-of-time distribution and a three-link parallel-serial network, while earlier studies are often conducted for the two-link parallel network with discretised value-of-time distributions. Anderson et al. (1992) systematically consider production differentiation in the transportation system with discrete choice models.

3.0 The Agent-Based Model: User Choices

In order to explore product differentiation on congested networks with heterogeneous users, the proposed agent-based approach requires the development of several component models, considering user choices (demand model), pricing choices, and capacity choices, respectively. This section describes the demand model and user choices regarding trip frequencies, destinations, and routes. Section 4 presents the models of pricing and capacity choices by public and private road authorities, followed by the analysis of production differentiation in subsequent sections.

3.1 Overview of the demand model

Trip frequency choices, and to some extent destination choices, are often represented by OD-specific predefined (inverse) demand functions in
road-pricing studies. Route choices are modelled after user equilibrium assignment principles. On a complex network, this approach may become problematic, as there are substitutional effects between destinations and the OD-specific demand functions are interdependent and no longer separable. The concept of ‘generalised travel cost’ allows the consideration of toll and travel time in traffic assignment, but does not provide an algorithm that assigns heterogeneous users with distinct values of time to alternative routes. Building on a previously developed agent-based approach for travel demand analysis (Zhang and Levinson, 2004), we develop a more advanced demand model in this paper that considers trips frequency, destination, and route choices by heterogeneous users. The term ‘agent-based’ implies that decision-makers have individual behavioural rules, possess learning capabilities, interact with each other, and adjust their behaviours adaptively over time.

There are several main assumptions in the proposed model. First of all, residential locations of all users are assumed to be fixed for simplicity and there are no land use changes. This may result in prediction biases, especially when the model is applied to produce long-term estimates due to the well-known land use–transportation interactions. This fixed land use, and therefore fixed overall travel demand, allows the model to achieve a long-term system equilibrium more easily. However, future research on long-term transportation network dynamics should strive to relax this assumption and use integrated land use–transportation models. Over a long period of time, the impact of transportation investments on land use could be significant. Any land-use changes will in turn influence subsequent transportation network dynamics. Each traveller in the model is assumed to have a predetermined travel budget, which is a simplified representation of the underlying trade-off between reducing travel cost and reaching premium activity locations. Two types of learning are considered in the model: learning from doing, and learning from information exchange. All information exchanges in the model occur exclusively and locally at nodes in the transportation systems, which is structurally appealing for model development but does not directly capture global information sharing (for example, through mass media). These and other minor model assumptions are further discussed and justified in the following sections.

3.2 Agents
3.2.1 Traveller
There are three types of agent in this agent-based travel demand model: traveller, node, and arc. Each ‘traveller’ agent represents a road user in the real world. A traveller is characterised by residential location, travel
budget ($u$), value of time ($v$), and perception threshold ($T$). The number of travellers residing at each residential location is limited by the available housing space at the location. Travel budget determines how far a traveller is willing to travel for a specific activity, an important factor in destination choice. Value of time provides travellers impetus to choose routes with different travel time and toll combinations. Perception threshold reflects the individuals’ inertia to make behavioural changes, measured by expected reductions in travel costs. These four traveller attributes are all fixed and drawn from predetermined distributions.

One of the two goals for each traveller is to find an activity opportunity that is located just far enough from the residential location such that the travel budget is exhausted. The evidence that travellers maintain commuting travel times, daily overall travel times, and generalised commuting costs over time is well documented in the travel behaviour literature (see Mokhtarian and Chen, 2004, for a review). It is conceivable that travellers may have a threshold beyond which they will not travel. The use of a travel budget in the proposed model is a simplification of the travellers’ underlying desire both to reduce travel cost and to reach premium activity locations. The actual destination choice usually represents a trade-off between travel cost and the quality of the activity location. Since detailed information regarding the quality of destinations and of how individuals evaluate destination quality is not readily available in most areas, we use the travel budget to represent the consequence of this underlying decision process. A traveller who cannot find a satisfactory destination within the travel budget forgoes the trip. The second traveller goal is to identify the route to the chosen destination with the lowest generalised cost. Travellers attempt to achieve this goal through learning-by-doing during personal explorations and learning from information exchanges with other travellers.

3.2.2 Node
In order to maintain a parsimonious model structure, each ‘node’ agent represents three identities in the real world:

- a vertex of the physical road network where roads (arcs) are connected;
- a centroid where residents and activity opportunities are located;
- a place where travellers can exchange information and learn from each other.

In reality, travellers learn destinations and routes from other travellers in many different ways. For instance, such information exchange can occur in offices, neighbourhood parks, restaurants, shopping centres, or through
phone conversations and emails. Information exchange at nodes should be a reasonable approximation of the real-world information-sharing process because the probability of travellers meeting increases with the overlap between parts of the network that they cover in their trips.

Travellers clearly cannot remember all possible paths between origins and destinations. Each node in the model therefore only stores the information of the $K$ shortest travel-time paths and $K$ lowest-toll paths from every other node to itself. This is necessary for users with different values of time to find the paths with the lowest general travel cost. Information exchange occurs when a traveller visits a node during the traveller’s destination and route search process. The travellers can provide the updated shortest-path information to nodes visited later. The node can deliver the updated shortest-path information to subsequent visiting travellers. This distributed learning mechanism does not guarantee the identification of the true shortest paths. However, it does reflect the limitation of real-world travellers in route learning. Simulation results in this study show that travellers are more likely to find the true shortest paths if the network is smaller (fewer alternative routes) and if the total number of travellers in the network is larger (more learning opportunities).

3.2.3 Arc
An ‘arc’ agent represents a directional roadway segment. Each arc is labeled with its origin node, destination node, capacity ($F$), free-flow travel time ($t_0$), and toll rate ($\tau$), and ownership status ($O$). The congested arc travel time ($t$) and generalised arc travel cost ($c$) can be derived from flow–cost functions. We adopt a simple representation of the supply-side of the transportation network, and employ the standard BPR function. Although our agent-based structure can readily accommodate a microsimulation dynamic network supply model, we choose to focus in this paper on route assignment with completely heterogeneous users that in itself represents a significant advance from existing traffic assignment procedures.

3.3 Behavioural rules and interactions
This subsection describes how aggregate demand patterns can be derived from the behaviours of and interactions among the three types of aforementioned agent. Given an initial residential location, each traveller examines activity opportunities by exploring the surrounding nodes, starting from the origin. The probability of moving to a specific node in each search step is proportional to the amount of remaining activity opportunities located at that node. During this initial exploration process, travellers accumulate their network knowledge, a form of learning-by-doing. The
probabilistic nature of the search process and the presence of a large number of travellers ensure a set of diversified initial route choices. In addition, an avoidance mechanism in the model guarantees that travellers do not visit the same node twice. This eliminates circular routes in the destination search, and also ensures that travellers will consider nodes further away from the origins, albeit with a smaller probability. A traveller selects a destination node once the personal travel budget is reached, or decides not to travel at all if no destination node previously-visited satisfies the travel budget. This initial destination choice algorithm is equivalent to a stochastic microscopic implementation of the intervening opportunities model (Stoufer, 1940; Haynes and Fotheringham, 1984) with a travel budget. Due to congestion effects and as learning continues, a traveller may need to adjust the destination and/or route choices after the initial destination and route are found.

When a traveller learns a new route, the traveller must determine whether or not to switch to the new route. The two alternative paths are evaluated based on the traveller’s value of time, perception threshold, path travel time, and path tolls. The route-switching rule is defined below:

\[
p = \begin{cases} 
\frac{e^{-(s/b)\gamma - T}}{e^{-(s/b)\gamma - T} + e^{-0.5\gamma T}} & \text{if } b > T, \\
0 & \text{otherwise,}
\end{cases}
\]

where \( p \) is the probability of switching to the new route; \( b \) is the actual benefit of switching route measured in dollars; \( T \) is the traveller’s perception threshold measured in dollars; and \( \gamma \) and \( s \) are the parameters defining the shape of the probability curve. A larger \( \gamma \) implies that the probability drops faster when the benefit reduces; \( s \) is a scale parameter that defines the overall tendency for travellers to switch routes.

When travellers switch routes, arc flows and travel times are updated according to the arc cost function. Consequently, the cost of all paths stored at nodes and learned by travellers would also change, which may trigger further route switching. A route choice equilibrium is achieved if no traveller, given the information available and the fixed value of time, has incentives to change routes. This is similar to the ‘behavioral user equilibrium’ concept in Zhang (2007). However, this model differs from Zhang (2007) in its implementation of route search and choice rules. If travellers can no longer satisfy their travel budgets by changing routes alone, they would then adjust their destination choices. When destination changes are necessary, travellers investigate the nodes adjacent to their current destinations in order to reduce their travel costs. A traveller may need to change destinations more than once to satisfy the travel budget.
When all travellers either are satisfied with their destinations or cancel their trips because a satisfactory destination cannot be found, a \textit{destination-trip frequency choice equilibrium} is achieved.

The combined route–destination-trip frequency choice equilibrium can be derived for a particular transportation network with the route choice equilibrium and the destination-frequency choice equilibrium simulated iteratively. The convergence can be measured by the maximum or average origin–destination demand difference (or other typical network convergence indicators) between consecutive simulation iterations. This final equilibrium represents the result of an implied game played by all travellers, where they exhibit both cooperative and non-cooperative behaviours. Travellers cooperate by sharing network information with each other. However, they only consider their own benefits when making individual destination and routing decisions.

3.4 Model calibration

We derived a set of model parameters for the Sioux Falls network which contains twenty-four nodes, seventy-six links, and 33,640 peak-hour trips as seen in the previous traffic assignment literature (for example, LeBlanc, 1975). The travel budget distribution is calibrated against observed trip length data. A log-normal distribution with a mean of 35.4 min and a standard deviation of 13.4 min is obtained from the calibration (see Figure 1). In theory, a distribution for the value of time may be derived from stated or

![Figure 1](Received and Modelled Travel Time Distributions on the Sioux-Falls Network)
revealed preference survey data and mixed discrete choice models. However, we do not have access to such data in the Sioux Falls area. Based on the shape of typical income distributions and the general correlation between income and value of time, we consider in the base scenario a log-normal value-of-time distribution with both the mean and standard deviation being US$15 per hour.

Coefficients in the route-switching rule (equation (1)) and travellers’ perception thresholds influence the route choice equilibrium in the agent-based model. Since the Sioux Falls network is a relatively small network, travellers should be able to identify the shortest paths even when the network is congested. We therefore calibrate the route choice coefficients against a standard user equilibrium traffic assignment pattern by enforcing uniform values of time for all travellers in the agent-based model. The final calibrated coefficients are $\gamma = 1$, $T = $0.10, and $s = 2$. The mean (maximum) link flow difference between the agent-based model and the user equilibrium model is 2.4 percent (6.4 per cent) under this set of route choice coefficients.

4.0 The Agent-Based Model: Price and Capacity Choices

In response to user choices, road operators determine whether or not to charge tolls, set the appropriate toll levels, and make capacity investment decisions. A pubic road agent is assumed to maximise social welfare given applicable political constraints. A private road agent maximises profits, given applicable regulatory constraints.

4.1 Public roads

Users of existing public roads pay for fuel taxes, vehicle sales and registration taxes, and driver’s license fees, and in some areas, general taxes for travel. These taxes produce an equivalent toll of around 2.5 cents per kilometre under the following conditions:

- 8.5 kilometres per litre average vehicle fuel efficiency;
- 10.5 cents per litre combined federal and state fuel taxes;
- fuel taxes constitute half of the total road preservation, maintenance, and improvement funds (this percentage is roughly consistent with the current road financing practices in the USA).

The current prevailing tax rate, instead of the socially optimal one, is adopted for the analysis. It should also be noted that users of private toll
roads will also pay fuel taxes in addition to the private tolls, resulting in
double charges on private road users.

The public authority allocates the pooled tax revenues and allows
cross-subsidisation among public roads. Empirical evidence shows that
public road authorities employ ranking systems to prioritise preservation,
maintenance, and capacity investment projects (Montes de Oca and
Levinson, 2006). The ranking systems are often based on a higher-level
allocation of funds between different types of expenditure, lower-level
project scores rated by an expert panel, and possibly some formula-based
mechanism that considers geographic equity. In the USA, public-road
authorities give priority to road preservation and maintenance activities,
and allocate the remaining funds for capacity expansion. We assume in
the model that revenue generated from road users is appropriated to first
defray road maintenance costs. The remaining revenue, if any, will then
be allocated to capacity expansion projects with the highest benefit–cost
ratios until revenue exhaustion. The method for computing the benefit–
cost ratios of expanding each road in the network is the same as that in
Zhang and Levinson (2005). Two Cobb–Douglas functions are estimated
in previous studies for capacity expansion and road-maintenance costs
respectively (Levinson and Karamalaputi, 2003; Zhang and Levinson,
2005), and adopted herein:

Capacity expansion cost ($K$):

$$K^i_a = \phi(l_a)^{\alpha_1}(F^i_a)^{\alpha_2}(F^{i+1}_a - F^i_a)^{\alpha_3}.  \quad (2a)$$

Maintenance cost ($M$):

$$M^i_a = \mu(l_a)^{\beta_1}(F^i_a)^{\beta_2}.  \quad (2b)$$

In the cost functions, $a$ is the index of roads and $i$ is the index of time
periods. Kilometres of roadway construction ($l$), additional capacity
added ($F^{i+1} - F^i$), existing road capacity ($F^i$), and technology factors
($\phi, \mu$) are variables. It has been shown that road-construction cost depends
on the length of the roadway, current roadway type (for example, freeway,
arterial streets that can be captured by the existing level of road capacity),
and the amount of capacity expansion (Levinson and Karamalaputi, 2003).
Empirical evidence on maintenance cost functions at the link level is an
understudied area in transportation economics. In theory, it should
depend on the length of the road, the type of the road, and possibly on
traffic volumes, as is corroborated by Paterson and Archondo-Callao
(1991) and recognised by the USA FHWA Highway Economic Require-
ment System (HERS).
4.2 Private roads

Previous economic analysis of private roads often makes strong assumptions about the availability and reliability of the demand information, and addresses the problems of profit-maximising toll and capacity on stylised networks by mathematical programming techniques. In contrast, we derive the price and capacity choices of private roads by examining the information actually available to private roads and by allowing private roads to adjust their decisions over time as more information becomes available.

Cooperation between private roads are assumed away, and each private road is only interested in its own profit in our analysis. Private roads maximise short-term profits by setting the appropriate tolls given the current capacity levels. The profit-maximising toll depends on travellers’ demand elasticities with respect to tolls, which depend on all substitutional and complementary effects in the network. To estimate accurately the demand elasticities on private roads for the current and future years on complex networks is difficult, due to intrinsic demand uncertainties, network complexity, and data availability for forecasting. Under these circumstances, private roads can better achieve their profit objectives by learning demand responses adaptively as they accumulate information on historical tolls, the resulting traffic flows, and profit levels. It is therefore assumed in the model that private roads employ price (toll) and quantity (flow) information in the previous time periods to estimate the underlying demand curves using line-fitting techniques. Simulation results in Zhang and Levinson (2005) show that the demand curves on toll roads are approximately linear when the majority of the roads on a grid network are untolled, and closely resemble power functions when toll roads are in the majority. However, to what extent their findings are applicable to general networks should be tested empirically in future research.

The following describes the detailed procedure for setting private road tolls. First, a rolling-horizon toll-profit function is empirically derived based on historical toll and profit information. The underlying demand curve on any private roads may shift due to toll adjustments and capacity expansions on other roads. Therefore, an intelligent private road should routinely update the toll-profit function to maximise profit. At each decision point, if the most recent toll-profit parabola has a maximal profit point (reverse U shape), the corresponding profit-maximising toll can be directly identified. If a maximal profit point does not exist on the parabola (U shape), the private roads should reduce or increase their current toll by a certain amount (assumed to be 50 per cent; reducing the toll if the current toll is on the left-hand side of the U curve, and increasing the toll if on the right-hand side). A real-world example of this type of adaptive toll-adjustment process is available. When the private Dulles
Greenway in Washington, DC opened in 1995, the one-way toll was US$1.75 per vehicle based on the demand estimates at the time, later reduced to US$1.00, owing to lower-than-expected levels of travel demand (an anonymous reviewer points out that this is primarily due to a downturn in the economy, leading to slower land development along the corridor). The toll was increased again in 1997 to US$1.15 to improve profit, followed by yet another toll increase in 2004 (FHWA 2006).

A private road expands its capacity when the additional capacity investment promises an investment return higher than that of all other investment opportunities available to the private investor \( r_0 \), assumed to be 6 per cent annually. The life-cycle cost of capacity expansion can be computed from equation (2) and a standard discounting process. However, the expected long-term investment return or profit cannot be estimated with much certainty as other roads in the network may also expand capacity and change tolls, therefore shifting future demand curves for the private road currently considering capacity expansion. We develop the following heuristic procedure for any private road to make capacity investment decisions under uncertainty:

\( \text{Step 1} \) Assume that the capacity and tolls on all other roads will remain unchanged throughout the thirty-year planning horizon; this implies that the current demand elasticity will hold in the future.

\( \text{Step 2} \) Compute travel-time savings for users of the particular private road that can result from a specific feasible amount of capacity expansion \( \Delta K \) (adding one more lane, or two more lanes, and so on).

\( \text{Step 3} \) Convert the travel-time savings on the private road following capacity expansion to a monetary value \( (\Delta \pi \text{-time}) \) based on the weighted average value of time of all travellers currently using the private road.

\( \text{Step 4} \) Increase the toll on the private road by \( \Delta \pi \) such that the total number of users on the private road remains unchanged.

\( \text{Step 5} \) Compute the annual additional profit gain throughout the planning horizon resulting from the increased toll on the link \( (\Delta \pi \text{-toll}) \).

\( \text{Step 6} \) Let \( \Delta \pi = \max (\Delta \pi \text{-time}, \Delta \pi \text{-toll}) \), which is the estimated total profit from the additional capacity.

\( \text{Step 7} \) Compute the annual return of capacity investment for the current capacity expansion scenario. Let the maximum annual return of all possible capacity investment scenarios be \( r^* \) and the corresponding capacity addition be \( \Delta K^* \).

\( \text{Step 8} \) The private road expands capacity by \( \Delta K^* \) if \( r^* > r_0 \); otherwise, there is no capacity expansion.

This heuristic provides a conservative estimate of the return on capacity investment because it only considers profit gains for private roads in two
extreme scenarios: the profit from increased traffic with tolls unchanged ($\Delta \pi_{-\text{time}}$), and the profit from increased toll only with higher levels of service ($\Delta \pi_{-\text{toll}}$).

The agent-based model of user, price, and capacity choices is now completely developed and ready for the subsequent analysis on product differentiation. It is important to recognise that these choices are made on dissimilar timescales by various decision-makers. In this paper, we assume that capacity expansion and pricing decisions are made once every year (or once in each simulated time period) by both public and private roads, which corresponds to the current fiscal-year and regulatory practices. Road users in the model can adjust their trip frequencies, destinations, and route choices multiple times in each year, but an overall travel demand equilibrium is reached once a year following annual price and capacity changes.

5.0 Product Differentiation on Congested Networks

The agent-based model is applicable to a variety of network economic analyses on complex networks with heterogeneous users, including alternative road-pricing schemes, investment policies, and ownership structures. The analysis and simulation results presented in this section both demonstrate the agent-based approach, and explore the evolution and impact of product differentiation on congested networks.

We create a mixed-ownership network by adding a parallel private toll road to each of the untolled seventy-six public roads in the Sioux-Falls network. The resulting test network has twenty-four nodes and 152 arcs. The total number of desired daily trips is 336,400, although certain trips may be cancelled due to unsuccessful destination searches. The agent-based model simulates only one peak hour per day. The hourly statistics are then converted into daily and annual statistics. The base-case network is heavily congested, as the average volume-to-capacity ratio under the assumption of no tolls on all roads is 1.41.

5.1 Definitions and measures of product differentiation

There are two levels of product differentiation: product differentiation of routes (path differentiation), and product differentiation of destinations (space differentiation). Path differentiation occurs when certain routes between origins and destinations are reserved for a specific group of users due to high (low) prices and good (poor) levels of service. When all routes reaching a particular destination can be afforded by only a specific
group of users, that destination becomes a place exclusively serving that particular user group, which results in an extreme case of space differentiation. In the present agent-based model, users are differentiated by their value of time (or by income). We use income and value-of-time interchangeably in the following discussion, recognising that these two characteristics are not perfectly correlated. It is also possible to label users in the model with other socioeconomic or demographic characteristics such as age, gender, and ethnicity.

In order to measure path differentiation, let us consider all users \( q \) between a particular OD pair. When a standard user equilibrium determines the route assignment and travel time is the only route attribute, all \( q \) users should experience exactly the same travel time, and pay zero toll. The level of path differentiation should be zero numerically in this case. However, when toll roads are present in the network, users with higher values of time are more likely to use the toll roads that operate at lower levels of congestion than their untolled counterparts. We propose two measures of these price and level-of-service differences to assess the level of path differentiation. The Gini coefficient of concentration (Gini, 1936) has been used by economists to analyse income inequality. A Gini coefficient of zero indicates perfect equality (the case of standard user equilibrium with no tolls), and one indicates perfect inequality (a single user suffers all the delays, or a single user pays all the tolls). It can be calculated as the ‘relative mean difference’; that is, the mean of the travel time (toll) difference between every pair of users, divided by the mean travel time (toll) (Glasser, 1962):

\[
G_t = \frac{\sum_{m=1}^{q} \sum_{n=1}^{q} |t_m - t_n|}{2q \sum_{m=1}^{q} t_m},
\]

\[
G_{\tau} = \frac{\sum_{m=1}^{q} \sum_{n=1}^{q} |\tau_m - \tau_n|}{2q \sum_{m=1}^{q} \tau_m},
\]

(3a)

(3b)

where \( G_t \) is the Gini coefficient of travel time inequality (Time Gini); \( G_{\tau} \) is the Gini coefficient of toll inequality (Toll Gini); \( t \) is the travel time; \( \tau \) is the toll, and \( m, n \) are indices of users between the OD pair.

The second measure of path differentiation requires the division of all users between an OD pair into several groups by value of time (VOT). We identify three VOT groups in this paper: high, medium, and low. The ratio of the average travel time of the high-VOT group to that of the
low-VOT group \((R_t)\) reveals the degree to which roads with higher levels of service are reserved for the high-VOT group. Similarly, the ratio of the average toll paid by the low-VOT group to that paid by the high-VOT group \((R_t)\) is also a good measure of path differentiation. Both ratios should be between zero and one, with zero implying maximum path differentiation, and one meaning no path differentiation. Unfortunately, this is just the opposite of the Gini coefficient. In order to minimise confusion, we use \((1 - R_t)\) and \((1 - R_t)\) as the second group of measures of path differentiation. For the remainder of the paper, \((1 - R_t)\) will be referred to as the Time Ratio, and \((1 - R_t)\) the Toll Ratio. In order to aggregate OD-level path differentiation measures to a network-wide measure, we employ the weighted average of the OD-level measures with the OD flows being the weights.

Space differentiation at a destination may be computed as the weighted mean of path differentiation from all origins. A more intuitive method is directly to examine the distribution of values of time among all users arriving at that destination. For instance, a destination may only serve users with high values of time because all paths reaching the destination are heavily tolled. In this sense, toll roads can potentially exacerbate the level of urban segregation by values of time. We measure the level of space differentiation also by a Gini coefficient:

\[
G_s = \frac{\sum_{m=1}^{Q} \sum_{n=1}^{Q} |VOT_m - VOT_n|}{2Q \sum_{m=1}^{Q} VOT_m},
\]

where \(G_s\) is the Gini coefficient of space differentiation (Space Gini); VOT is the value of time; \(Q\) is the total number of users arriving at the destination; and \(m, n\) are the indices of users arriving at the destination.

### 5.2 Welfare measures

Measuring annual producers’ surplus on congested networks is relatively straightforward, being the total annual revenue minus the amortised construction and maintenance costs. When there is a single origin–destination pair on the network, the area underneath the fixed OD demand curve and above the line representing the actual price paid provides a measure of the consumers’ surplus. However, when there exist multiple interdependent OD pairs on a complex network, a good measure of consumer’s surplus is less obvious. Total travel time savings on the network does not represent the full user benefits because travel-time savings (and cost reductions) on the network can result in longer trips to more attractive destinations and
mode shifts. We adopt the following measure of consumers’ surplus in this paper:

\[
CS_y^i = \sum_{v=1}^{V} \left[ \frac{1}{2} \cdot \left( \overline{p}_{l,v}^0 - \overline{p}_{l,v}^r \right) \cdot (q_{l,v}^0 + q_{l,v}^r) \right],
\]

(4a)

\[
\overline{p}_{l,v}^r = \frac{\sum_{j=1}^{q_{l,v}^y} (\tau_{l,v}^j + VOT_j \cdot t_{l,v}^j)}{q_{l,v}^y},
\]

(4b)

where \( CS_y^i \) is the consumers’ surplus of user group \( i \) (high, medium, and low-income groups); \( y \) is the index of simulation periods; \( v \) is the index of OD pairs; \( V \) is the total number of OD pairs in the network; \( \overline{p}_{l,v}^0 \) is the average generalised cost paid by users in group \( i \) with OD pair \( v \); \( q_{l,v}^y \) is the total number of users in group \( i \) with OD pair \( v \); \( j \) is the index of users in group \( i \) with OD pair \( v \); \( t \) is the travel time; \( \tau \) is the toll; and \( VOT_j \) is the value of time of user \( j \).

This measure of consumers’ surplus is clearly sensitive to travel-time savings and toll reductions. It is also sensitive to destination choices and travel distances because the selection of a new destination by a user causes \( q \) to decrease by one for the old OD pair, and to increase by one for the new OD pair; that is, although each traveller in our model has a fixed travel budget, a destination change would still register a change in consumers’ surplus according to equation (4). The consumer welfare measured defined in equation (4) implies fixed demand curves for all OD pairs, guaranteed by the fixed land-use assumption in this paper. However, when applied to scenarios with variable land use, this measure may produce biased welfare estimates. Further research may also develop more sophisticated measures of user benefits on complex networks that better account for welfare changes associated with the quality of destinations. Measures not considering destination quality tend to underestimate the welfare gains resulting from transportation investment.

5.3 Base-case results
Both price and capacity competition is allowed between public and private roads in the based case. System dynamics are simulated over 100 iterations for the network to reach a stable pattern (see Figure 2). The private sector reacts to the user demand for more road capacity and improved level of service much faster than the public sector, as seen by the sharp increase of total lane-kilometres of private roads in early iterations. It should be noted that the above finding is under the assumption that the public sector does not borrow from either the private sector or other revenue
sources to improve public roads. In the real world, government agencies may borrow money and finance much-needed new transportation facilities through various kinds of bond that are paid back by future revenue streams. The model results also show that private roads can learn the demand elasticities over time and gradually increase the toll to the maximum level of US$0.35 per kilometre. Eventually, public roads are able to expand their capacity with available funding to the point when the total tax revenue just defrays the total maintenance cost. Over time, capacity expansions on public roads force private roads to reduce their tolls to about US$0.17 per kilometre at the long-run network equilibrium. Lower private tolls then lead to capacity reductions on certain private roads because their annual toll revenues fall short of annual maintenance costs.

Figure 3 plots the level of production differentiation with measures developed in Section 5.1. It is evident that toll-related measures of path differentiation (Toll Gini and Toll Ratio) are much higher than time-related measures (Time Gini and Time Ratio). This indicates that toll-road users accept a large premium of toll charges in exchange for a disproportionately small reduction in travel time. For example, at the long-run network equilibrium, the high-income group on average pays about four times more than the low-income group for just a 19 per cent reduction in travel time (Time Ratio = 0.19). All four measures of
path differentiation provide consistent results, and therefore could be used interchangeably on this test network for policy comparison purposes. The profiles of the path differentiation measures closely resemble the profile of private tolls in Figure 2, which is expected. Interestingly, the level of space differentiation increases monotonically without displaying a decreasing trend following the toll reductions. This suggests that once a destination has been established as primarily serving high- or low-income groups, that status is unlikely to be changed by a small toll reduction. Nevertheless, toll roads clearly have increased space differentiation on the test network. In other words, by making certain destinations more or less amenable to specific income groups, road pricing could exacerbate urban segregation by income.

In the base case, all income groups have benefited from the increased level of product differentiation due to price and capacity competition (see Figure 4). Not surprisingly, the high-income group reaps the majority of consumer benefits, while the low-income group barely experiences a positive change. Since the three income groups are divided into equal shares of the total population, the above finding also holds on a per capita basis. The overall net social benefit is quite stable, following the quick capacity expansion on private roads in the first several iterations. However, a redistribution of welfare gains between private roads and users is obvious during the simulation period. The driving force of this
welfare redistribution is the slow but steady capacity expansion on public roads. However, this redistribution regarded as favourable by public policy makers comes at a significant cost of $2.85 billion including construction and maintenance costs. This cost may be avoided with proper regulation on the pricing and/or capacity investment decisions of private roads. Although the agent-based model can be readily applied to regulatory analysis, we leave that for future research to keep the focus on product differentiation.

The agent-based structure of the network economics model allows one to track the decision-making process and status of each user and each road authority, which provides certain system statistics that are not available from traditional equilibrium analysis. These statistics could be important for various financial and policy analysis tasks. Figure 5 provides two examples: the total number of users priced off the network, and the percentage of profitable private roads. The first statistic is valuable for the equity considerations of network financing policies. The second statistic can provide important inputs to the investment decisions of private roads (and toll roads in general), and the analysis of optimal regulation. Nearly 40 per cent of the private roads will not be profitable at the equilibrium. This is due to the overinvestment period at the beginning of the simulation, triggered by both true demand for new road facilities and the ignorance of
the collective influence of competitors’ investment decisions (similar to a stock-market or housing-market bubble). The model assumes that every private road has to maintain a minimum level of road capacity to sustain network connectivity. In reality, private road authorities that lose money will most probably abandon or sell the roads unless certain regulatory restrictions exist.

5.4 Variations on two themes
Various parameters in the base case could be allowed to vary, which would produce a large number of test scenarios for various policy analysis tasks. We choose to alter the base case on two themes:

- the type of competition allowed;
- the degree of user heterogeneity.

5.4.1 Different types of competition
The base case incorporates both price and capacity competition. We create two additional scenarios with only one of the two types of competition allowed respectively (see Table 1). When capacity expansion is prohibited, public roads have practically no instrument to compete with private toll roads because they do not have freedom to adjust prices either. Therefore, private roads are able to impose on users a heavy toll as high as US$1.02
per kilometre. The level of product differentiation is astoundingly high in this scenario with price competition only. According to the time (0.37) and toll ratios (0.94), toll-road users pay seventeen times more than users of untolled roads to save only 37 per cent of travel time. Clearly, only a very small portion of the users with very high incomes are willing to use the toll roads, while others are either priced off the toll roads, or off the network entirely. It is interesting to note that the medium-income group takes the hardest hit, although all three income groups are hurt by the high tolls. Some high-income users could benefit from the over-priced toll roads as shown by their willingness to pay. Many low-income users have marginal welfare gains from their trips initially, and they can minimise their losses due to high tolls or heavy congestion simply by cancelling their trips. Consequently, the medium-income group does not have the most to lose, but loses the most. Verhoef and Small (2004) have found similar unfavourable results for medium-income travellers in a pay-lane network when a second-best pricing scheme is imposed.

The next scenario under the theme of competition types only allows capacity competition, and enforces fixed prices on both private (US$0.17 per kilometre, same as the average toll in the base case) and public roads (US$0.025 per kilometre). The welfare measures with only capacity competition are almost identical to the base case, indicating that in the base case a larger share of potential benefits are to be reaped from capacity changes as opposed to pricing. This also suggests that properly restricting

### Table 1

**Network Statistics with Different Types of Competition**

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Base case</th>
<th>Price competition</th>
<th>Capacity competition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average private road toll (US$/km)</td>
<td>0.17</td>
<td>1.02</td>
<td>0.17 (fixed)</td>
</tr>
<tr>
<td>Lane-km of private roads</td>
<td>1,085</td>
<td>605 (fixed)</td>
<td>994</td>
</tr>
<tr>
<td>Lane-km of public roads</td>
<td>1,178</td>
<td>605 (fixed)</td>
<td>989</td>
</tr>
<tr>
<td>Path differentiation: Time Gini</td>
<td>0.04</td>
<td>0.11</td>
<td>0.03</td>
</tr>
<tr>
<td>Path differentiation: Toll Gini</td>
<td>0.32</td>
<td>0.45</td>
<td>0.26</td>
</tr>
<tr>
<td>Path differentiation: Time Ratio</td>
<td>0.18</td>
<td>0.37</td>
<td>0.13</td>
</tr>
<tr>
<td>Path differentiation: Toll Ratio</td>
<td>0.78</td>
<td>0.94</td>
<td>0.61</td>
</tr>
<tr>
<td>Space differentiation: Space Gini</td>
<td>0.17</td>
<td>0.25</td>
<td>0.16</td>
</tr>
<tr>
<td>Private road profit (million US$)</td>
<td>73</td>
<td>480</td>
<td>65</td>
</tr>
<tr>
<td>Consumers’ surplus: high income</td>
<td>381</td>
<td>–67</td>
<td>389</td>
</tr>
<tr>
<td>Consumers’ surplus: medium income</td>
<td>197</td>
<td>–127</td>
<td>203</td>
</tr>
<tr>
<td>Consumers’ surplus: low income</td>
<td>15</td>
<td>–39</td>
<td>16</td>
</tr>
<tr>
<td>Net social benefit</td>
<td>666</td>
<td>247</td>
<td>673</td>
</tr>
</tbody>
</table>
the pricing freedom of toll roads has minimum impact on both the overall net social benefit and the distribution of welfare gains. This finding supports road authorities to adopt simple and transparent tolling policies, such as network-wide flat distance-based tolls. Implementing flat tolls could be a practical way to limit economically inefficient price competition on a road network with competing toll roads. With fixed toll rates, the same level of net social benefit can actually be achieved with less overall capacity expansion than the base case. The level of path and space differentiation is also lower with fixed prices.

5.4.2 Different levels of user heterogeneity
Previous studies on small single-OD networks have discovered the important role of user heterogeneity in the welfare analysis of toll roads (Arnott et al., 1992; de Palma, 1992; Schmanske, 1993; Small and Yan, 2001; Verhoef and Small, 2004; Xin and Levinson, 2006). The base case assumes the mean and the standard deviation of the log-normal value-of-time distribution to be both US$15 per hour. Four additional scenarios are obtained with the standard deviation ranging from US$0 to US$30 per hour in Table 2. As the degree of user heterogeneity increases, we can observe that:

1. Private roads charge higher tolls because the high-income users are willing to pay more for the same level of service.
2. The high-income group enjoys increased consumers’ surplus.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Network Statistics with Different Degrees of User Heterogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOT Standard deviation (US$/hr)</td>
<td>0</td>
</tr>
<tr>
<td>Statistics</td>
<td></td>
</tr>
<tr>
<td>Average private road toll (US$/km)</td>
<td>0.16</td>
</tr>
<tr>
<td>Lane-km of private roads</td>
<td>1,226</td>
</tr>
<tr>
<td>Lane-km of public roads</td>
<td>1,184</td>
</tr>
<tr>
<td>Path differentiation: Time Gini</td>
<td>0</td>
</tr>
<tr>
<td>Path differentiation: Toll Gini</td>
<td>0</td>
</tr>
<tr>
<td>Path differentiation: Time Ratio</td>
<td>0.15</td>
</tr>
<tr>
<td>Path differentiation: Toll Ratio</td>
<td>0.65</td>
</tr>
<tr>
<td>Space differentiation: Space Gini</td>
<td>0</td>
</tr>
<tr>
<td>Private road profit (million US$)</td>
<td>71</td>
</tr>
<tr>
<td>Consumers’ surplus: high income</td>
<td>257</td>
</tr>
<tr>
<td>Consumers’ surplus: medium income</td>
<td>257</td>
</tr>
<tr>
<td>Consumers’ surplus: low income</td>
<td>257</td>
</tr>
<tr>
<td>Net social benefit</td>
<td>842</td>
</tr>
</tbody>
</table>
3. The low-income group sees consumers’ surplus decreased and will eventually suffer welfare losses.

Changes in the level of user heterogeneity appear to have insignificant influence on the capacity expansion decisions by either private or public road authorities. In general, higher levels of product differentiation are observed with higher degrees of user heterogeneity; but this trend does not always hold (note the Time Gini and Time Ratio measures). When the total net social benefit is examined, a ‘worst’ value-of-time distribution from the welfare perspective exists (Mean = Std. Dev. = US$15 per hour). The lack of monotonicity (and therefore predictability) in welfare changes suggests that knowledge of the actual VOT distribution critical in the design and the full-impact analysis of toll road policies. This lack of monotonicity in welfare changes can have structural reasons. When there is no variation in VOT (this implies homogeneous users), private roads have to build a great deal of capacity and charge relatively low tolls to compete with public roads. Users clearly benefit from this type of competition. When VOT variations are present in the system, the product differentiation process will discriminate against those with low VOT and benefit those with high VOT. If the level of VOT variation is relatively low, the additional benefit for the high-VOT users may not be sufficient to recover the loss endured by the low-VOT users. This can lead to a net loss in total welfare. However, if the level of VOT variation is very high, the benefit for high-VOT users exceeds the absolute loss for low-VOT users, leading to increased social welfare.

6.0 Conclusions

This paper develops a novel and completely agent-based approach for network economics analysis, and applies this approach to analyse price competition, capacity choice, and product differentiation on congested networks. This evolutionary paradigm complements the existing network equilibrium methods, and brings several important advantages. First, the decision-making process, behavioural adjustments, and actual experience of each user and each supplier can be tracked in the agent-based approach, which makes it especially capable of analysing the distributitional effects of network management and financing policies. Considering user heterogeneity is also straightforward in the evolutionary approach. Second, the agent-based approach is applicable to large real-world networks. Analysts can use this tool to design and evaluate policy scenarios with consideration of the comprehensive spatial and temporal effects of these policies.
Product differentiation is inevitable in the presence of price and/or capacity competition, as seen in the airline, rail, transit industries, and increasingly, on road networks. This research defines two types of product differentiation for networks: path differentiation and space differentiation, and develops several practical measures. Results show that price competition is a more significant source of product differentiation than capacity competition. When toll roads seek maximum revenues, users with higher values of time could pay much higher tolls than those with lower values of time for a disproportionately small amount of time savings. While in most cases users with the lowest values of time harvest the least benefit (or suffer the most loss) from road pricing and investment decisions, users with the medium values of time can take the hardest hit in some cases. Not surprisingly, users with the highest values of time always benefit the most from product differentiation in all tested scenarios.

Another finding is that the relationship between net social benefit and user heterogeneity is not monotonic on a complex network with tolled and untolled roads. There exists a socially least optimal level of user heterogeneity that corresponds to the lowest level of total social welfare. Higher degrees of user heterogeneity do not always result in higher social benefits of toll roads. It is therefore important to collect information on the actual distribution of users’ values of time for the welfare analysis of private or public toll roads.

References


