Modeling the Cost Structure of Public Transit Firms: Scale Economies and Functional Form (#09-3435)

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Abstract

This study analyzes the cost structure of a set of medium-large, U.S. urban public transit firms, with special attention to the issue of scale economies in the production of transit services. Short and long-run costs are modeled using two of the more commonly-employed functional forms in the urban transit cost function literature, the Cobb-Douglas and transcendental logarithmic (or translog) cost functions. The more general specification of the translog cost function also allows for some investigation of production structure relating to factor demand and substitution. The data set employed in the econometric estimation of the cost functions is a pooled time series of 23 firms of similar size observed over the period from 1996 to 2003. Results indicate that estimates of scale economies are sensitive to the choice of an output measure. Cobb-Douglas estimates show evidence of short-run returns to density and size, but long-run diseconomies of scale. Short-run translog estimates indicate evidence of stronger of returns to density and size, raising questions about the source of increasing returns. Lastly, while the more general translog specification allows for a more flexible production structure, its estimation constraints and large number of parameters make it more difficult to properly estimate and interpret.

Theory and Estimation

In estimating a cost function for urban transit firms, we use the fact that a *dual* relation exists between a firm's production and cost functions (Braeutigam 1999). As is shown elsewhere (McCarthy 2001; Karlaftis and McCarthy 2002), a firm's cost function summarizes all relevant economic information contained in its production function. If we define a cost function as: C = C(**p**,y;γ), where *C* represents a firm's costs, ***p*** is a vector of input prices, *y* represents a firm's output (or set of outputs), and *γ* represents the existing state of technology, then the functional form for the cost function follows from the assumptions about the underlying production structure of the firm.

Cobb-Douglas Cost Function

$$\ln C_V = a_0 + \sum_{i=1}^{y} a_i \ln y_i + \sum_{i=1}^{J} b_i \ln p_i + \varepsilon$$

Translog Cost Function

While the Cobb-Douglas and Leontief functional forms could be employed to answer fairly straightforward questions about the cost and production of urban transit firms, they also entailed some important restrictions that limited their applicability. As noted previously, Leontief production technology assumes both constant returns to scale and substitution elasticities of zero for all inputs. Also, while Cobb-Douglas cost functions allow for nonconstant returns to scale and input substitution, the structure of these cost functions restrict substitution elasticities to unity and assume constant estimates of returns to scale over the entire range of output levels (Braeutigam 1999).

The translog cost function represents a second-order Taylor series approximation to an arbitrary cost function about its mean value. This specification allows for non-linear effects in each of the input factors, as well as interactions between input factors in the cost function, represented by quadratic and cross-product terms in the cost function (Berndt 1991).

The translog (short run) variable cost function can be written as:

$$\begin{aligned}\ln C_V &= a_0 + a_y \ln y + \sum_{i=1}^{J} a_i \ln p_i + a_N \ln N + a_k \ln k + \sum_{i=1}^{J} g_{iy} \ln p_i \ln y \\ &+ \sum_{i=1}^{J} g_{iN} \ln p_i \ln N + \sum_{i=1}^{J} g_{ik} \ln p_i k + g_{yN} \ln y \ln N + g_{yk} \ln y \ln k + g_{Nk} \ln N \ln k \\ &+ \frac{1}{2}\sum_{i=1}^{J}\sum_{i=1}^{J} g_{ij} \ln p_i \ln p_j + \frac{1}{2} g_{yy} (\ln y)^2 + \frac{1}{2} g_{NN} (\ln N)^2 + \frac{1}{2} g_{kk} (\ln k)^2 \\ &+ b_{tti} \ln TTI + b_{pb} \ln Peak + t_i + e, \quad i,j = 1,\ldots,J\end{aligned}$$

While the translog cost function could be estimated directly, Berndt (1991) argues that gains in efficiency can be realized by simultaneously estimating the associated optimal, cost-minimizing input demand equations. These input demand equations are obtained by differentiating the cost function with respect to factor prices and employing Shephard's Lemma:

$$\frac{\partial \ln C}{\partial \ln p_i} = s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \gamma_{iy} \ln y + \gamma_{iN} N + \varepsilon_i, \quad i = 1,\ldots,J$$

These efficiency gains in estimation can be important, especially for smaller data sets, since the translog cost function usually contains a large number of estimable parameters.

Defining and Measuring Returns to Scale

While economies of scale are largely a long-run concept, it is possible to specify different measures of short-run returns to scale (Karlaftis and McCarthy 2002). The variable cost function provides measures of returns to network size (RTSZ) and capacity utilization (RTCU), where the fixed factor of production is defined as rolling stock. Including network size in the model, we can further distinguish between economies of size and traffic density. Using a "produced" measure of output (e.g. vehicle miles), the elasticity of cost with respect to output gives an estimate of economies of size. Introducing a "consumed" measure of output (e.g. passenger boardings) yields an estimate of economies of traffic density (Berechman 1993).

Data and Variables

In order to estimate the cost functions and associated share equations described previously, a panel data set was assembled using publicly-available financial and operating data on U.S. transit firms contained in the National Transit Database (NTD), which is maintained by the Federal Transit Administration (FTA). The data set contains observations on 23 firms over 8 years (1996 to 2003) for a total of 184 observations. The group of firms included in the sample are similar to previous work by Karlaftis and McCarthy (2002). Using a large sample of firms from 1986 to 1994, they used hierarchical clustering, a data reduction technique, in order to identify six groups of transit systems with similar operating characteristics. The firms chosen for the present study are those falling into the second-largest size group, including all but the five or six largest bus systems in the U.S.

Results

Table 1: Summary Data for Transit Firms

Variable	Mean	S.D.
Operating Characteristics		
Operating cost ('000s $)	137,116	45,428
Number of vehicles	730	244
Gallons of fuel ('000s)	6,998	2,333
Vehicle miles ('000s)	26,304	8,218
Passengers ('000s)	62,037	20,025
Factor Input Variables		
Price of labor ($)	26.90	4.91
Price of fuel ($)	1.23	0.15
Price of materials ($)	39,250	15,797
Share of labor ($)	0.76	0.07
Share of fuel ($)	0.04	0.01
Share of materials ($)	0.20	0.07

Table 2: Variable Labels and Definitions

Variable	Description
LN C_V	LN of variable cost (C_V)
LN C_T	LN of total cost
LN Pl	LN of price of labor
LN Pm	LN of price of materials
LN Pf	LN of price of fuel
LN k	LN of fixed capital factor (number of buses)
LN Pk	LN of price of capital
LN VM	LN of annual vehicle miles
LN Pax	LN of passenger boardings
LN Pl-Pm	LN of labor-materials interaction
LN Pl-Pf	LN of labor-fuel interaction
LN Pm-Pf	LN of materials-fuel interaction
LN Pl-Pk	LN of labor-capital interaction
LN Pm-Pk	LN of materials-capital interaction
LN Pf-Pk	LN of fuel-capital interaction
LN Pl-k	LN of labor-fixed capital interaction
LN Pm-k	LN of materials-fixed capital factor interaction
LN Pf-k	LN of fuel-fixed capital factor interaction
LN Pl-VM	LN of labor-vehicle miles interaction
LN Pm-VM	LN of materials-vehicle miles interaction
LN Pk-VM	LN of vehicle miles-capital interaction
LN Pf-VM	LN of fuel-vehicle miles interaction
LN k-VM	LN of vehicle miles-fixed capital factor interaction
LN Pl-pax	LN of labor-passengers interaction
LN Pm-pax	LN of materials-passengers interaction
LN Pf-pax	LN of fuel-passengers interaction
LN Pk-pax	LN of capital-passengers interaction
LN k-pax	LN of fixed capital factor-passengers interaction
LN Pl-N	LN of labor-network size interaction
LN Pm-N	LN of materials-network size interaction
LN Pf-N	LN of fuel-network size interaction
LN Pk-N	LN of capital-network size interaction
LN k-N	LN of fixed capital factor-network size interaction
LN VM-N	LN of vehicle miles-network size interaction
LN pax-N	LN of passengers-network size interaction
1/2 Pl-Pl	1/2 * LN of price of labor squared
1/2 Pm-Pm	1/2 * LN of price of materials squared
1/2 Pf-Pf	1/2 * LN of price of fuel squared
1/2 k-k	1/2 * LN of fixed capital factor squared
1/2 Pk-Pk	1/2 * LN of price of capital squared
1/2 VM-VM	1/2 * LN of vehicle-miles squared
1/2 pax-pax	1/2 * LN of passengers squared
1/2 N-N	1/2 * LN of network size squared
LN N	LN of network size
LN TTI	LN of Travel Time Index
LN P/B	LN of peak-to-base ratio
Time Trend	Time trend variable

Table 3: Short and Long-Run Cobb-Douglas Cost Functions

	Model 1[a]		Model 2[b]		Model 3[c]		Model 4[d]	
Variable	Coefficient	SE	Coefficient	SE	Coefficient	SE	Coefficient	SE
LN Pl	0.72 (***)	0.06	0.45 (***)	0.07	0.78 (***)	0.06	0.47 (***)	0.10
LN Pm	0.26 (***)	0.03	0.28 (***)	0.03	0.20 (***)	0.03	0.19 (***)	0.04
LN Pf	0.04	0.09	0.10	0.10	0.10	0.12	0.17	0.17
LN k	0.46 (***)	0.07	0.79 (***)	0.06				
LN VM	0.78 (***)	0.08			1.14 (***)	0.05		
LN Route Miles	-0.21 (***)	0.03	0.03	0.03	-0.20 (***)	0.03	0.35 (***)	0.03
LN TTI	-0.01	0.10	-0.20 (*)	0.12	-0.01	0.11	-0.37 (**)	0.16
LN P/B	-0.15 (***)	0.05	-0.24 (***)	0.06	0.01	0.05	0.02	0.08
Time Trend	0.00	0.00	0.01	0.01	-0.01	0.01	0.00	0.01
LN Pk					-0.14	0.28	0.15	0.40
LN Pax			0.26 (***)	0.04			0.54 (***)	0.05
Constant	-1.24	1.10	4.03 (***)	0.87	-4.09 (***)	0.93	2.01 (*)	1.14
Adjusted R^2	0.88		0.86		0.86		0.71	
D-W Statistic	2.14		2.46		2.28		2.57	
N = 184								

[a] Short-run, variable cost function with vehicle miles as output
[b] Short-run, variable cost function with passenger boardings as output
[c] Long-run, total cost function with vehicle miles as output
[d] Long-run, total cost function with passenger boardings as output
Notes: Pi = Price of input i, f = fuel, k = fixed capital factor (rolling stock), l = labor, m = materials, VM = vehicle miles, pax = passenger boardings, N = Network size
(*) indicates significance at $p < 0.10$ level, (**) indicates significance at $p < 0.05$ level, and (***) indicates significance at $p < 0.01$ level

Table 4: Translog Variable Cost Functions

	Model 1[a]		Model 2[b]		Model 3[c]	
Variable	Coefficient	SE	Coefficient	SE	Coefficient	SE
LN Pl	0.81	1.32	0.69	1.330	0.90	0.92
LN Pm	1.69	1.21	-0.64	1.11	0.10	0.92
LN Pf	-1.50	1.27	0.94	1.24	0.00	0.00
LN k	5.37 (***)	1.52	4.12 (***)	1.49	8.62 (***)	2.41
LN VM	2.60	1.60			-11.55 (***)	3.88
LN Pax			0.77	1.34		
LN Pl-Pm	-0.65 (***)	0.19	-0.06	0.23	-0.76 (***)	0.10
LN Pl-Pf	0.65 (***)	0.23	0.17	0.26	0.90 (***)	0.33
LN Pm-Pf	0.00	0.23	-0.12	0.25	0.00	0.12
LN Pl-k	-0.22	0.18	-0.66 (***)	0.17	-0.06	0.12
LN Pm-k	0.01	0.12	-0.08	0.09	0.24 (***)	0.08
LN Pf-k	-0.12	0.22	0.05	0.23	-0.30 (***)	0.11
LN Pl-VM	-0.12	0.15			0.29 (***)	0.11
LN Pm-VM	-0.23 (*)	0.12			-0.33 (***)	0.09
LN Pf-VM	0.35 (**)	0.15			0.04	0.07
LN k-VM	-0.04	0.21			-0.60 (***)	0.23
LN Pl-pax			0.11	0.11		
LN Pm-pax			-0.12 (**)	0.06		
LN Pf-pax			0.01	0.12		
LN k-pax			0.20 (*)	0.12		
LN Pl-N	0.32 (***)	0.12	0.19 (*)	0.11	-0.39 (***)	0.06
LN Pm-N	0.05	0.07	0.02	0.06	0.11 (***)	0.04
LN Pf-N	-0.37 (***)	0.13	-0.21	0.13	0.28 (***)	0.07
LN Pk-N						
LN k-N	-0.01	0.02	-0.01	0.02		
LN VM-N	0.13	0.08			0.15 (*)	0.09
LN pax-N			0.09 (*)	0.05		
1/2 Pl-Pl	0.84 (***)	0.23	0.17	0.26	-0.24	0.17
1/2 Pm-Pm	0.31 (***)	0.07	0.35 (***)	0.06	0.44 (***)	0.04
1/2 Pf-Pf	-0.41	0.32	0.13	0.37	-0.33	0.26
1/2 k-k	-0.43	0.34	-0.62 (***)	0.19	0.27	0.25
1/2 VM-VM	-0.09	0.16			1.00 (***)	0.32
1/2 pax-pax			-0.10	0.10		
1/2 N-N	0.00	0.05	0.02	0.03	-0.08 (**)	0.04
LN N	-2.09 (**)	1.02	-1.68	1.04	-3.36 (***)	1.21
LN TTI	0.07	0.07	0.01	0.08	-0.05	0.04
LN P/B	-0.04	0.03	-0.08 (***)	0.03	0.02	0.02
Time Trend	0.00	0.01	0.01	0.01	0.02 (**)	0.01
Constant	-28.76 (**)	12.58	-1.84	13.00	91.72 (***)	24.39
Adjusted R^2	0.92		0.90		0.67	
N = 184						

[a] Short-run, variable cost function with vehicle miles as output
[b] Short-run, variable cost function with passenger boardings as output
[c] Same specification as model 1 with additional constraint that $0 < a_i < 1$
Notes: Pi = Price of input i, f = fuel, k = fixed capital factor (rolling stock), l = labor, m = materials, VM = vehicle miles, pax = passenger boardings, N = Network size

Returns to Size and Capacity Utilization

Table 5 provides estimates of returns to size and capacity utilization for the translog cost function, along with the Cobb-Douglas cost function estimates for comparison.

Table 5: Returns to Scale Estimates for Translog and Cobb-Douglas Cost Functions

Model	RTCU[a]	RTSZ[b]	SRAC[c]	SRMC[d]
Short-run				
Cobb- Vehicle Miles	0.43	0.22	5.21	4.05
Cobb-Douglas Passengers	0.72	0.74	2.21	0.57
Translog Vehicle Miles	1.01	0.73	5.21	1.39
Translog Passengers	0.91	0.82	2.21	0.40

Model	RTSZ	LRAC	LRMC
Long-run			
Cobb- Vehicle Miles	-0.14	5.93	6.76
Cobb-Douglas Passengers	0.46	2.51	1.35

Factor Demand and Substitution

In addition to estimates of returns to scale, the translog cost function can provide estimates of factor price elasticities and substitution elasticities. The substitution elasticities can be calculated for the translog cost function using the Allen-Uzawa partial elasticities of substitution (Berndt 1991). For the translog specification, these are:

$$\sigma_{ij} = \frac{\gamma_{ij} + s_i s_j}{s_i s_j}, \quad i,j = 1,\ldots J, \ \text{for } i \neq j,$$

$$\sigma_{ii} = \frac{\gamma_{ii} + s_i^2 - s_i}{s_i^2}, \quad i = 1,\ldots J$$

and the own price elasticity of demand is given by $(e_{ii}) = \sigma_{ii} s_l$, where s_l is the share of input *i* in costs. Estimates of the demand and substitution elasticities are shown in Table 6. The top half of Table 6 presents the estimates for Model 1, in which input price coefficients were not constrained to take values between zero and one, but all other restrictions were imposed to ensure linear homogeneity.

Table 6: Own-Price Elasticties and Elasticity of Substitution Estimates from Translog Cost Function

Own-Price Elasticities			Elasticties of Substitution		
Partially-Constrained Estimates, Vehicle-Miles Output					
e_{ll}	e_{mm}	e_{ff}	σ_{lm}	σ_{lf}	σ_{mf}
-0.08	-0.06	0.29	0.46	0.52	0.01
Fully-Constrained Estimates, Vehicle-Miles Output					
e_{ll}	e_{mm}	e_{ff}	σ_{lm}	σ_{lf}	σ_{mf}
-0.41	N/A	0.51	N/A	-7.44	N/A

Note: l = labor, m = materials, f = fuel