The Physics of Traffic Congestion and Road Pricing in Transportation Planning

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What is Traffic Congestion?

Congestion occurs when Traffic Demand > Roadway Capacity

Congestion results in queueing, slower travel times, (perhaps) reduced throughput, schedule delay, and wasted resources.

Two obvious solutions:

(1) Increase capacity
(2) Reduce demand
What is Capacity?

Capacity is the maximum number of vehicles that can move past a point (bottleneck) in a given time (e.g. veh/hour). This is determined by the nature of the vehicles and by the size of the bottleneck.

How to increase capacity?

(1) Increase size of bottleneck (constrained by cost of land, construction, topology, etc.)

(2) Better organize vehicles to move with less “friction”. Constrained by abilities of drivers and cars. Intelligent Transportation Systems aim to do this.
Capacity

Is a property of both the road and the drivers/vehicles.

People drive (ideally) to minimize travel time while ensuring safety, i.e. ensure a safe stopping distance, given perception reaction time of individuals (this varies), vehicle braking time (which depends on vehicle and road conditions). Gaps between vehicles (especially large gaps) are lost capacity.

People are not deterministic in the way that rocks are. From here out however, we will assume they are, and that there is a fixed capacity.
Demand

Number of Trips (trip generation)
Destination of Trips (trip distribution models)
Mode of Trips (mode choice models)
Time of Trips (departure time choice models, bottleneck model)
Route of Trips (network route choice models)

All effected by the time and cost of travel
If you increase capacity, and reduce travel times, you will attract traffic. This must be considered in any real analysis.
## Types of Pricing

<table>
<thead>
<tr>
<th>Coarse Fine</th>
<th>Average cost</th>
<th>Profit-Maximizing*</th>
<th>Marginal cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>General with Uniform Links</td>
<td>Gas Tax</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Facility-specific</td>
<td>Public Toll Roads</td>
<td>Private Toll Roads (Public Toll Roads?)</td>
<td>Microfoundations Model Bottleneck Model</td>
</tr>
<tr>
<td>High Occupancy Toll Lanes</td>
<td>I-394</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area-Based</td>
<td>London</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cordon</td>
<td></td>
<td>Oslo, Bergen, Trondheim</td>
<td>Cordon Pricing Model</td>
</tr>
<tr>
<td>General with Differentiated Links</td>
<td>Mileage Based User Fees</td>
<td></td>
<td>Ideal</td>
</tr>
</tbody>
</table>

Note: * Regulated (price-capped) Profit-Maximizing

Tuesday, March 16, 2010
Wardrop’s Principles

User Equilibrium: The journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route. (like a Nash Equilibrium, developed in parallel)

System Optimal: At equilibrium the average journey time is minimum.

System Travel Times:
S.O. < U.E.

With pricing S.O. = U.E.
S.O. What?

The Price of Anarchy is the ratio of U.E. to S.O. time,

\[
\text{Price of Anarchy} \geq 1
\]

But is it \(>> 1\)?

If the Price of Anarchy is small, congestion is more a temporal than spatial phenomenon.
Traffic Assignment Differences
7:30 - 8:30 AM
Twin Cities, Minnesota

User Equilibrium Flow Rate
subtracted from
System Optimal Flow Rate

VEHICLE COUNTS
-2414 - -2000  -24 - 0
-1999 - -1000  1 - 25
-999 - -500    26 - 50
-499 - -200    51 - 100
-199 - -100    101 - 200
-99 - -50      201 - 500
-49 - -25      501 - 956
<table>
<thead>
<tr>
<th></th>
<th>User Equilibrium</th>
<th>System Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle km</td>
<td>9326447</td>
<td>9373185</td>
</tr>
<tr>
<td>Vehicle Hours</td>
<td>150964</td>
<td>148389</td>
</tr>
<tr>
<td>Speed (km h⁻¹)</td>
<td>61.8</td>
<td>63.2</td>
</tr>
<tr>
<td>Trips</td>
<td>632232</td>
<td></td>
</tr>
<tr>
<td>Time Savings</td>
<td></td>
<td>1.7%</td>
</tr>
</tbody>
</table>
Objective of Research

• To build simplest model that explains congestion phenomenon and shows implications of congestion pricing.

• Use game theory to illustrate ideas, informed by structure of congestion problems
  – simultaneous arrival;
  – arrival rate > service flow;
  – first-in, first-out queueing,
  – delay cost,
  – schedule delay cost
Two boats, One lock
Game Theory Assumptions

Actors are instrumentally rational
(actors express preferences and act to satisfy them)

Common knowledge of rationality
(each actor knows each other actor is instrumentally rational, and so on)

Consistent alignment of beliefs
(each actor, given same information and circumstances, would make same choice)

Actors have perfect knowledge
Two-Player Congestion Game

Penalty for Early Arrival (E), Late Arrival (L), Delayed (D)

Each vehicle has option of departing (from home) early (e), departing on-time (o), or departing (l)

If two vehicles depart from home at the same time, they will arrive at the queue at the same time and there will be congestion. One vehicle will depart the queue (arrive at work) in that time slot, one vehicle will depart the queue in the next time slot.
Congesting Strategies

• If both individuals depart early (a strategy pair we denote as $ee$), one will arrive early and one will be delayed but arrive on-time. We can say that each individual has a 50% chance of being early or being delayed.

• If both individuals depart on-time (strategy $oo$), one will arrive on-time and one will be delayed and arrive late. Each individual has a 50% chance of being delayed and being late.

• If both individuals depart late (strategy $ll$), one will arrive late and one will be delayed and arrive very late. Each individual has a 50% chance of being delayed and being very late.
### Payoff Matrix

<table>
<thead>
<tr>
<th>Vehicle 1</th>
<th>Vehicle 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Early</strong></td>
<td></td>
</tr>
<tr>
<td>Early</td>
<td>[0.5*(E+D), 0.5*(E+D)]</td>
</tr>
<tr>
<td>On-time</td>
<td>[0, E]</td>
</tr>
<tr>
<td>Late</td>
<td>[L, E]</td>
</tr>
</tbody>
</table>

Note: [Payout for Vehicle 1, Payout for Vehicle 2]

Objective to Minimize Own Payout, S.t. others doing same
**Example: (3,1,4)**

<table>
<thead>
<tr>
<th></th>
<th>Early</th>
<th>On-time</th>
<th>Late</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Early</td>
<td>[2,2]</td>
<td>[3,0]</td>
<td>[3,4]</td>
</tr>
<tr>
<td>On-time</td>
<td>[0,3]</td>
<td>[2.5, 2.5]</td>
<td>* [0,4]</td>
</tr>
<tr>
<td>Late</td>
<td>[4,3]</td>
<td>[4,0]</td>
<td>[6.5, 6.5]</td>
</tr>
</tbody>
</table>

Note: * Indicates Nash Equilibrium

*Italicics* indicates social welfare maximizing solution
# Payoff matrix with congestion pricing

<table>
<thead>
<tr>
<th>Vehicle 1 (Early)</th>
<th>Vehicle 2 (Early)</th>
<th>Vehicle 2 (On-time)</th>
<th>Vehicle 2 (Late)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early</td>
<td>$[0.5\times(E+D)+\tau_e,$</td>
<td>$[E,$</td>
<td>$[E,$</td>
</tr>
<tr>
<td></td>
<td>$0.5\times(E+D) + \tau_e]$</td>
<td>$0]$</td>
<td>$E]$</td>
</tr>
<tr>
<td>On-time</td>
<td>$[0,$</td>
<td>$[0.5\times(L+D)+\tau_o,$</td>
<td>$[0,$</td>
</tr>
<tr>
<td></td>
<td>$E]$</td>
<td>$0.5\times(L+D) + \tau_o]$</td>
<td>$L]$</td>
</tr>
<tr>
<td>Late</td>
<td>$[L,$</td>
<td>$[L,$</td>
<td>$[L+0.5\times(L+D)+\tau_i,$</td>
</tr>
<tr>
<td></td>
<td>$E]$</td>
<td>$0]$</td>
<td>$L+0.5\times(L+D) + \tau_i]$</td>
</tr>
</tbody>
</table>
What are the proper prices?

- Normally use marginal cost pricing
  \[ MC = \frac{\partial TC}{\partial Q} \]
- But Total Costs (TC) are discrete, so we use incremental cost pricing
  \[ IC = \frac{\Delta TC}{\Delta Q} \]
- Total Costs include both delay costs as well as schedule delay costs.
  \[ \tau_o = \tau_l = 0.5*(L+D) \]
  \[ \tau_e = 0.5*(D). \]
Subtleties

• Vehicles may affect other vehicles by causing them to change behavior.

• Total costs do not include these “pecuniary” externalities such as displacement in time, just what the cost would be for that choice, given the other person is there, compared with the cost for that choice if one player were not there.

• You can’t blame departing early on the other player.
### Example (3,1,4) with Congestion Prices

<table>
<thead>
<tr>
<th>$E, D, L$</th>
<th>Vehicle 1</th>
<th>Vehicle 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,1,4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Early</td>
<td>On-time</td>
</tr>
<tr>
<td>Early</td>
<td>[2.5,2.5]</td>
<td>[3,0]*</td>
</tr>
<tr>
<td>On-time</td>
<td>[0,3]*</td>
<td>[5,5]</td>
</tr>
<tr>
<td>Late</td>
<td>[4,3]</td>
<td>[4,0]</td>
</tr>
</tbody>
</table>
## Equilibrium Conditions

<table>
<thead>
<tr>
<th>State</th>
<th>No Pricing</th>
<th>With Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>ee</td>
<td>$E=0$ and $D=0$</td>
<td>$E=0$ and $D=0$</td>
</tr>
<tr>
<td>eo, oe</td>
<td>$E \leq 0.5(L+D)$ and $E \leq L$</td>
<td>$E \leq L$</td>
</tr>
<tr>
<td>el, le</td>
<td>$E=0$ and $L=0$</td>
<td>$E=0$ and $L=0$</td>
</tr>
<tr>
<td>oo</td>
<td>$D \leq L$ and $0.5(L+D) \leq E$</td>
<td>$L \leq E$ and $D=0$</td>
</tr>
<tr>
<td>ol, lo</td>
<td>$L \leq E$ and $L \leq D$</td>
<td>$L \leq E$</td>
</tr>
<tr>
<td>ll</td>
<td>$L=0$ and $D=0$</td>
<td>$L=0$ and $D=0$</td>
</tr>
</tbody>
</table>

*Note:* Assume penalties $E$, $L$, $D \geq 0
Summary

• Presented a simple (the simplest?) model of congestion and pricing.
• A new way of viewing congestion and pricing in the context of game theory.
• Illustrates the effectiveness of moving equilibria from individually to socially optimal solutions.
• Extensions: empirical estimates of E, D, L; risk; uncertainty and stochastic behavior; simulations with more players.
Network-based marginal cost models are not consistent with the physics of traffic

Bottleneck models cannot describe the overcrowding of networks

At what mobility level a city should operate?
Motivation

\( I \rightarrow (Z) \gamma \rightarrow A \)

(Vickrey, 1969)

\( I(t) \rightarrow A(t) \)

late arrivals

early arrivals

(Vickrey, 1969)
Vickrey’s Bottleneck model as above holds for links, BUT: Cities’ output decreases with level of congestion.

(Vickrey, 1969)
Recent findings

Accumulation
(Number of vehicles in the network)

Production
(Veh-km traveled/u.t)

- A Macroscopic Fundamental Diagram (MFD) exists
- Trip completions / Network flow \( \approx \) Constant

(Geroliminis & Daganzo, 2007, 2008)
Accumulation
(Number of vehicles in the network)

Recent findings

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(Geroliminis & Daganzo, 2007, 2008)
Recent findings

Accumulation
(Number of vehicles in the network)

- A Macroscopic Fundamental Diagram (MFD) exists
- Trip completions / Network flow ≈ Constant

Production
(Veh-km traveled/ u.t.)

Output
(Number of trips ending/ u.t.)
Recent findings

- A Macroscopic Fundamental Diagram (MFD) exists
- Trip completions / Network flow \( \approx \) Constant

(Geroliminis & Daganzo, 2007, 2008)
MFD (Empirical data)

Average flow $q''$ (vhs/5min) vs. Average occupancy $o''$ (%)

Yokohama, Japan

10 km$^2$

(Geroliminis & Daganzo, 2008)
Network equilibrium (No-toll case)

Notation

$I(t)$: Inflow
$Z(t)$: Desired arrival rate
$A(t)$: Arrival rate
$w$: travel delay
$s(t)$: schedule delay
$n_0$: critical accumulation
$\tau$: trip time
Equilibrium Solution

\[ w(n(t)) \]

\[ t_0 \quad t_\mu \quad t_3 \quad \text{time} \]

\[ e/c_w \quad \triangledown l/c_w \]
**EQUILIBRIUM SOLUTION**

BUT: Demand Congestion interval \([t_1, t_2]\) is a subset of Actual Traffic Congestion interval \([t_0, t_3]\)

A constructive solution of \(t_0, t_3\) is described in the paper
**Fine toll to reduce congestion**

Savings (excluding schedule delay)

\[
\Delta S = \frac{1}{2} \left( \delta (notoll)^2 - \delta (\gamma)^2 \right) \frac{\gamma le}{l + e}
\]

\[
T(t) = \begin{cases} 
\frac{e}{c_w} (t - t_0) & \text{if } t_0 \leq t \leq t_\mu \\
-\frac{l}{c_w} (t - t_3) & \text{if } t_\mu \leq t \leq t_3 \\
0 & \text{all other times}
\end{cases}
\]
Policy implications: Equity vs. Reliability

Equity

Efficiency

Reliability
Conclusions

• Constructed parsimonious models of congestion on links, networks.
• Pricing reduces or eliminates congestion, maximizing throughput on networks given interdependence of supply and demand.
• Cordon tolls can be applied to real networks to reduce or eliminate deadweight loss of queueing, prioritize high value trips at times of scarce road capacity. Examples London, Singapore, Stockholm, Oslo.
Papers


Thank You

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http://nexus.umn.edu