UNIVERSITY OF MINNESOTA

This is to certify that I have examined this copy of a master's thesis by

Michael James Iacono

and have found that it is complete and satisfactory in all respects,
and that any and all revisions required by the final
examining committee have been made.

Dr. David M. Levinson

____

Name of Faculty Advisor

____

Signature of Faculty Advisor

____

Date

GRADUATE SCHOOL

Modeling the Cost Structure of Public Transit Firms: The Scale Economies Question and Alternate Functional Forms

A THESIS
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Michael James Iacono

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE

Dr. David M. Levinson, Advisor

July, 2006

ACKNOWLEDGEMENTS

I would like to acknowledge Dr. David Levinson and Dr. Gerard McCullough for reviewing and commenting on previous versions of this paper. Their thoughts and guidance helped to transform this work from a simple term paper to thesis project.

ABSTRACT

This study analyzes the cost structure of a set of medium-large, U.S. urban public transit firms, with special attention to the issue of scale economies in the production of transit services. Short and long-run costs are modeled using two of the more commonly-employed functional forms in the urban transit cost function literature, the Cobb-Douglas and transcendental logarithmic (or translog) cost functions. The more general specification of the translog cost function also allows for some investigation of production structure relating to factor demand and substitution. The data set employed in the econometric estimation of the cost functions is a pooled time series of 23 firms of similar size observed over the period from 1996 to 2003. Results indicate that estimates of scale economies are sensitive to the choice of an output measure. Cobb-Douglas estimates show evidence of short-run returns to density and size, but long-run diseconomies of scale. Short-run translog estimates indicate evidence of stronger of returns to density and size, raising questions about the source of increasing returns. Lastly, while the more general translog specification allows for a more flexible production structure, its estimation constraints and large number of parameters make it more difficult to properly estimate and interpret.

TABLE OF CONTENTS

LIST OF TABLES

1. INTRODUCTION

Interest has grown in recent decades in studying the production of urban transit services in metropolitan regions in the U.S. and elsewhere. Improvements in data availability and econometric techniques for studying the cost behavior of firms allow for more in-depth study of the production technology employed by public transit agencies. In particular, the development of cost functions with more theoretically appealing forms has stimulated interest in modeling public transit costs and production processes. The reasons for studying such cost functions are many. Among them are regulatory and pricing issues, labor and management issues, and investment decisions about the optimal mix of inputs to employ in the production process.

For many years, public transit firms in the U.S. were comprised largely of streetcar systems requiring large amounts of capital investment. Thus, economists and regulatory bodies were predisposed to treat them as natural monopolies and grant them exclusive franchises to operate in urban areas under the justification of economies of scale. Widespread replacement of streetcars with less capital-intensive bus systems from the 1920s through the 1960s brought renewed interest to the question of whether scale economies could continue to justify monopolistic operations, whether in the private or public sector. The prospect of privatization or the introduction of some other form of increased competition in transit service delivery requires knowledge of firms' cost behavior, particularly as related to scale of operations. Such knowledge can inform public sector operating authorities or regulatory bodies on issues of appropriate pricing policies or industry structure.

In addition to pricing, cost functions describing the economic structure of transit firms may offer information about the possibilities for dealing with rising input costs. Recent increases in the price of fuel as well as health care benefits (a major component of labor costs) have financially squeezed many public transit firms, requiring difficult and often unpopular decisions regarding fares and service levels. Models based on actual operating experience may provide evidence of opportunities for appropriate substitution of inputs.

The study of public transit cost and production may also yield insights into the ways that individual firms combine inputs in the production process. Knowledge of the shares of each input that firms employ, along with their production characteristics (cost complementarity, input substitutability), can suggest how inputs might be reorganized in order to lower costs. Furthermore, knowledge about the cost-minimizing mix of inputs may inform the debate over the merits of bus-based versus rail-based transit, each of which emphasizes investment in different inputs (essentially labor and capital).

In the literature on public transit cost function estimation, two types of functional form have become increasingly important, namely the Cobb-Douglas and transcendental logarithmic (or "translog") specifications. While both represent theoretically consistent models of the production process, they offer substantially different outcomes in terms of econometric estimation issues and useful model output. In this paper, the use of both types of model is illustrated using a pooled time series data of 23 medium-sized, U.S. public bus transit firms for the years 1996 through 2003. Each model is used to derive estimates of scale economies for bus transit operations. In addition, output from the translog cost function is used to calculate preliminary estimates of input demand and substitution elasticities. Some policy implications regarding costs, pricing, and investment are drawn from the model estimates, using the Twin Cities metropolitan region as an example. Finally, some concluding remarks are offered regarding the merits of each modeling approach and recent advancements in analytical techniques for studies of urban public transit cost and production.

2. STUDIES OF PUBLIC TRANSIT COST FUNCTIONS

Over the past half-century there have been numerous studies of the cost structure of public transit firms. Many of the studies, particularly the earlier efforts, were concerned chiefly with estimates of the presence and/or extent of scale economies in bus transit operations. The conceptual developments that followed in the 1970s and 1980s brought about the ability to pose and provide answers to additional questions concerning the demand for the input factors (labor, fuel, etc.) of production, the substitutability of these inputs, the correct functional form to describe production and costs, and the causes of productivity change in the bus transit industry over time. A summary description of many of these studies is contained in Table 1.

The earliest studies date to the 1970s, with many of the studies carried out in Great Britain[1]. These studies were conducted in response to proposals by local authorities to merge smaller municipal operations into significantly larger Passenger Transport Authorities, under the hypothesis that cost savings were available via consolidation of fleets and management functions. Most of these studies reported constant returns to scale.

[1] The one known study carried out prior to 1970 was Johnston's (1956) study of British urban bus operations.

Table 1: Previous Cost Function Studies

Authors	Database	Functional Form	Type of Function	Measure of Output	Scale Economies	Economies of Density	Economies of Capacity Utilization
Cross-Sectional Studies							
Johnston (1956)	UK	Linear, Log-linear	Short-run	VM[a]	Increasing		
			Long-run		Increasing		
Lee and Steedman (1970)	UK	Linear	Short-run	VM	Constant		
Wabe and Coles (1975)	UK	Linear	Short-run	VM	Decreasing		
			Long-run		Decreasing		
Williams (1979)	US	Cobb-Douglas	Short-run	VM			Increasing
			Long-run	VM	Increasing		
Viton (1981)	US	Translog	Short-run	VM		Increasing	Increasing
			Long-run	VM	Decreasing (large systems)		
Williams and Dalal (1981)	US	Translog	Long-run	VM	Increasing (large systems)		
					Decreasing (small systems)		
Obeng (1984)	US	Translog	Short-run	PM[b]		Increasing	
			Long-run	PM	Decreasing		
Obeng (1985)	US	Translog	Short-run	PM		Increasing	
			Long-run	PM	Decreasing		
Button and O'Donnell (1985)	UK	Translog	Long-run	Passengers, Revenue	Increasing (small systems)		

					Decreasing (large systems)		
de Rus (1990)	Spain	Translog	Long-run	VK[c]	Decreasing/ constant		
				Passengers	Increasing		
Karlaftis et al. (1999a)		Translog	Short-run	VM	Increasing (small, medium)	Increasing (medium, large)	
					Decreasing (large)	Decreasing (small/ suburban)	
				Passengers		Increasing	
Time series studies							
De Borger (1984)	Belgium	Translog	Short-run	VM		Decreasing	
Berechman and Giuliano (1984)	US	Translog	Long-run	VM	Decreasing		
				Passengers	Increasing		
Colburn and Talley (1992)	US	Translog	Long-run	Multi-output	Increasing		Increasing
Berechman (1993)	Israel	Translog	Long-run	Passengers	Increasing		
Karlaftis et al. (1999b)	US	Translog	Short-run	VM	Decreasing	Constant	
				Pax		Increasing	
Panel data studies							
Kumbhakar and Bhattacharyya (1996)	India	Translog	Short-run	PK[d]		Increasing	
Karlaftis and McCarthy (2002)	US	Translog	Short-run	VM			Increasing

[a] Vehicle miles

[b] Passenger miles

[c] Vehicle kilometers

[d] Passenger kilometers

Source: Karlaftis and McCarthy (2002), author's additions

Most of these early studies employed either linear or log-linear specifications for the functional form of the cost function. Little explanation was given regarding the production technology assumed or the technical relationship between production, output and costs. Thus, the results could only be viewed as tentative.

More recent studies of cost structure and scale economies have employed econometric specifications that link the concepts of production and cost, and exploit the duality between them. Drawing on previous empirical work in related industries (e.g. Nerlove (1963)), researchers began to estimate cost functions with more general forms, such as the Cobb-Douglas and transcendental logarithmic (or "translog") models. These cost functions represented an improvement in that they involved fewer restrictions on the substitution possibilities available to producers, and also in that estimates of scale economies were not restricted to be constant over the entire range of output[2]. Since 1980, the translog specification has largely been the favored functional form for modeling transit firms' cost structures.

Though many studies of the cost structure of urban transit operations have been undertaken in recent decades, there has been remarkably little consensus on the issue of scale economies (or other matters relating to production technology). In addition to assuming different production technologies and functional forms, researchers have employed different types of data sets (cross-sectional, time series, panel), different numbers of input factors (usually ranging between 2 and 4), different output measures, different time horizons (short versus long-run), and firms of dramatically different sizes. A recent study (Karlaftis and McCarthy 2002) attempted to synthesize several of these

[2] This is true of the translog specification, which includes second-order terms, but not of the Cobb-Douglas specification (or any simpler functional forms).

issues by estimating short-run, variable cost functions for a large panel data set, which was subsequently broken down into six groups with similar operating characteristics. By distinguishing between firms of various sizes, and hence limiting the heterogeneity bias that characterized some earlier cross-sectional studies, the authors were able to demonstrate that urban transit firms of different size were in fact characterized by different cost structures. This finding extended to the estimation of scale economies.

This paper adopts the approach of attempting to isolate a group of firms with similar size and production characteristics (medium to large U.S. bus transit firms) in order to investigate further the question of scale economies and production technology. Different functional forms are employed (Cobb-Douglas and translog), along with two different measures of output (vehicle-miles and passenger boardings), representing both produced and consumed outputs. The focus is primarily on short-run variable cost functions, though some long-run estimates are provided for comparison in certain instances.

3. THEORY AND ESTIMATION

In estimating a cost function for urban transit firms, we use the fact that a *dual* relation exists between a firm's production and cost functions (Braeutigam 1999). As is shown elsewhere (McCarthy 2001; Karlaftis and McCarthy 2002), a firm's cost function summarizes all relevant economic information contained in its production function. If we define a cost function as: $C = C(\mathbf{p},y;\gamma)$, where C represents a firm's costs, $\boldsymbol{p}$ is a vector of input prices, y represents a firm's output (or set of outputs), and γ represents the existing state of technology, then the functional form for the cost function follows from the assumptions about the underlying production structure of the firm.

For the purposes of this paper, a short-run variable cost function will be specified with three inputs (labor, fuel, and materials) and a fixed factor of production, representing a transit firm's rolling stock (buses). The cost function can be generalized to a long-run function by replacing the fixed factor of production with an input (capital), and specifying a factor input price.

3.1 Cobb-Douglas and Leontief Cost Functions

In the short run then, the firm's expenditures on operations can be described by the following equation:

$$E = P_L L + P_F F + P_M M \qquad 1)$$

where E represents expenditures, and P_L, P_F, and P_M represent input prices for labor, fuel, and materials, respectively. If we assume that the firm's production of an output (y) is described by Cobb-Douglas technology,

$$y = AL^a F^b M^c \qquad 2)$$

then we can describe the firm's objective as minimizing E (or alternatively cost, denoted by C), subject to its production function, here described by the Cobb-Douglas form. If the firm minimizes expenditures while producing any level of output y, then its costs will be given by:

$$C = By^{1/(a+b+c)}(P_L)^{a/(a+b+c)}(P_F)^{b/(a+b+c)}(P_M)^{c/(a+b+c)} \qquad 3)$$

where B is a combination of the constants in the production function, and a, b, and c are constants to be estimated. In order to convert this cost function into a more easilyestimated equation, we can transform the equation by taking natural logarithms and rewriting it as:

$$\ln C_V = a + b_1 \ln P_L + b_2 \ln P_F + b_3 \ln P_M + b_4 \ln y \qquad 4)$$

where $a = \ln B$, $b_1 = a/(a+b+c)$, $b_2 = b/(a+b+c)$, $b_3 = c/(a+b+c) = (1 - b_1 - b_2)$, and $b_4 = 1/(a+b+c)$. We use C_V in this case to denote the variable cost function. The variable cost function can also be written more compactly as:

$$\ln C_V = a_0 + \sum_{i=1}^{y} a_i \ln y_i + \sum_{i=1}^{J} b_i \ln p_i + \varepsilon \qquad 5)$$

Where p_i denotes the factor price for input i and b_i denotes the share for input i for i = 1…J inputs, and where ε is a classical disturbance term.

The Cobb-Douglas cost function has a couple of notable properties. First, it allows for nonconstant returns to scale, which makes it useful for testing the scale economies hypothesis[3]. Also, it allows for input substitutability, a feature not possible with more restrictive forms of production technology (McCarthy 2001).

An example of a more restrictive form of production technology is the Leontief (or fixed proportions) cost function. Unlike the Cobb-Douglas function, the Leontief cost function assumes fixed proportions of inputs, thus allowing no possibility for input substitution. In addition, the Leontief production function exhibits constant returns to scale, a likely reason that it has not been used extensively in studying urban transit cost functions. A firm producing with Leontief technology would have a cost function of the form:

---

[3] The Cobb-Douglas form has been employed to study bus transit cost functions by Williams (1979). Studies of rail transit costs in the U.S. using Cobb-Douglas cost functions include Pozdena and Merewitz (1978) and Viton (1980).

$$C = a_0 + \sum_{i=1}^{J} b_i p_i + \varepsilon \qquad 6)$$

where b and p are defined as before. Interestingly, many of the earlier studies of cost functions using linear or log-linear forms were assuming Leontief production technology, even though this was not explicitly stated.

3.2 The Translog Cost Function

While the Cobb-Douglas and Leontief functional forms could be employed to answer fairly straightforward questions about the cost and production of urban transit firms, they also entailed some important restrictions that limited their applicability. As noted previously, Leontief production technology assumes both constant returns to scale and substitution elasticities of zero for all inputs. Also, while Cobb-Douglas cost functions allow for nonconstant returns to scale and input substitution, the structure of these cost functions restrict substitution elasticities to unity and assume constant estimates of returns to scale over the entire range of output levels (Braeutigam 1999).

In order to relax these restrictions, econometric researchers developed various types of "flexible" cost functions, which were considerably more general than the two described above. Their flexibility derived from the fact that they place no *a priori* restrictions on input substitution or returns to scale (McCarthy 2001). One particularly popular flexible functional form, the transcendental logarithmic (or "translog") specification, was introduced by Christensen, Jorgenson, and Lau (1973). The translog cost function represented a second-order Taylor series approximation to an arbitrary cost function about its mean value. This specification allows for non-linear effects in each of the input factors, as well as interactions between input factors in the cost function, represented by quadratic and cross-product terms in the cost function (Berndt 1991).

In order to specify the functional form of the translog cost function, we can return to the general specification given earlier, C = C(**p**,y;γ). In order to estimate a short-run, variable cost function, we will assume that one factor of production (rolling stock, taken to be a measure of bus capital) is held fixed. We can denote this fixed factor *k*. Also, including a measure of network size, defined here as the number of route-miles over which a transit system operates (denoted as *N*), allows for estimates of network-held-constant output elasticities in the short run. Thus, we can express the short-run, variable cost function as $C_V = C_V(\mathbf{p},y,Z;k,\gamma)$, with *Z* representing a vector of variables relating to the market in which a firm operates. This characterization of the variable cost function leads to an equation of the form:

$$\begin{aligned}
\text{Ln } C_V = {} & a_0 + a_y \ln y + \sum_{i=1}^{J} a_i \ln p_i + a_N \ln N + a_k \ln k + \sum_{i=1}^{J} g_{iy} \ln p_i \ln y \\
& + \sum_{i=1}^{J} g_{iN} \ln p_i \ln N + \sum_{i=1}^{J} g_{ik} \ln p_i k + g_{yN} \ln y \ln N + g_{yk} \ln y \ln k + g_{Nk} \ln N \ln k \\
& + \tfrac{1}{2} \sum_{i=1}^{J} \sum_{i=1}^{J} g_{ij} \ln p_i \ln p_j + \tfrac{1}{2} g_{yy} (\ln y)^2 + \tfrac{1}{2} g_{NN} (\ln N)^2 + \tfrac{1}{2} g_{kk} (\ln k)^2 \\
& + b_{tti} \ln TTI + b_{pb} \ln Peak + t_i + e, \quad i,j = 1,\ldots,J \qquad 7)
\end{aligned}$$

where t_i is a time trend variable that takes on the value of 1,2,…n for each year (1996 to 2003) represented in the sample data, *TTI* is a variable representing the travel time index reported by the Texas Transportation Institute in their Annual Mobility Report[4], and *Peak* is a variable representing the ratio of vehicles in service during peak and off-peak periods. The peaking characteristics of transit systems are assumed to be influenced (at least partly) by exogenous demand, which, along with congestion levels, could be expected to influence operating costs. Berechman and Giuliano (1985) argue for the

[4] The travel time index is a measure of region-wide traffic congestion computed from aggregate data on traffic volumes and roadway capacity. It is defined as the ratio of congested (peak-period) travel time to travel time during free-flow conditions. For the derivation of this index, see Schrank and Lomax (2005).

inclusion of similar types of variables in order to account for different demand environments and production conditions. A long-run, total cost function can also be specified by replacing the fixed factor of production by a variable factor and including its input price.

3.3 Estimating the Cost Functions

The Cobb-Douglas cost function is relatively easy to estimate and interpret, making it a natural choice to describe the cost structure of transit firms. Since its functional form is log-linear, it can be estimated using ordinary least squares. The estimation of the translog cost function is considerably more complex.

While the translog cost function could be estimated directly, Berndt (1991) argues that gains in efficiency can be realized by simultaneously estimating the associated optimal, cost-minimizing input demand equations[5]. These input demand equations are obtained by differentiating the cost function with respect to factor prices and employing Shephard's Lemma:

$$\frac{\partial \ln C}{\partial \ln p_i} = s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \gamma_{iy} \ln y + \gamma_{iN} N + \varepsilon_i, \quad i = 1,\ldots,J. \qquad 8)$$

Taken together, the cost function and share equations form a system of equations that can be efficiently estimated using the method of seemingly unrelated regression equations (SURE), developed by Zellner (1962).

Additional restrictions must be imposed on the cost function in order to be consistent with cost-minimizing behavior. In order to ensure that the cost function is well

---

[5] These efficiency gains in estimation can be important, especially for smaller data sets, since the translog cost function usually contains a large number of estimable parameters.

behaved, it must be homogeneous of degree one (Berndt 1991). This implies the following restrictions:

$$\sum_{i=1}^{J} a_i = 1, g_{ij} = g_{ji} \forall i, j,$$

$$\sum_{i=1}^{J} g_{ij} = \sum_{j=1}^{J} g_{ji} = \sum_{i=1}^{J} g_{iy} = \sum_{i=1}^{J} g_{iN} = 0. \qquad 9)$$

Lastly, while the input factor shares should sum to one, the individual factor shares should also take on values between zero and one. One of the sets of estimates produced here will include additional restrictions on the parameter values in order to ensure this.

3.4 Defining and Measuring Returns to Scale

The concept of increasing returns to scale in bus transit is premised on the idea that bus systems maintain a certain level of fixed costs which, by their nature, are invariant to output. Thus, increasing output will increase costs by a less-than-constant amount. Button (1993) claims that one possible source is economies of vehicle size. Other studies have emphasized the use of fixed plant (e.g. vehicle storage facilities) or large vehicle fleets (Berechman and Giuliano 1985). Likewise, administrative staff, which comprise a significant share of costs, can be considered to be a fixed cost (McCarthy 2001)[6].

Regardless of the source, it is argued that the industry is characterized by a U-shaped cost curve, with costs declining over a range of output, then leveling off and rising in the range of decreasing returns to scale. Increasing returns to scale at low levels of output are interpreted as firms not being able to fully exploit their investments in fixed plant. Declining returns to scale at higher levels of output are interpreted in light of the

[6] This is true in the short run, though larger changes in output levels may require more labor inputs.

organizational complexity of large firms and limits to the utilization of capital inputs (Berechman and Giuliano 1985).

While economies of scale are largely a long-run concept, it is possible to specify different measures of short-run returns to scale (Karlaftis and McCarthy 2002). The variable cost function provides measures of returns to network size (RTSZ) and capacity utilization (RTCU), where the fixed factor of production is defined as rolling stock. Including network size in the model, we can further distinguish between economies of size and traffic density. Using a "produced" measure of output (e.g. vehicle miles), the elasticity of cost with respect to output gives an estimate of economies of size. Introducing a "consumed" measure of output (e.g. passenger boardings) yields an estimate of economies of traffic density (Berechman 1993). In either case, the estimate of returns to size is given by:

$$RTSZ = 1 - \frac{\partial \ln C}{\partial \ln y} \qquad 10)$$

Returns to capacity utilization measure the impact on costs from a proportional increase in output and network size, and are denoted as:

$$RTCU = 1 - \left(\frac{\partial \ln C}{\partial \ln y} + \frac{\partial \ln C}{\partial \ln N} \right) \qquad 11)$$

Measures of short-run average and marginal cost can also be derived from the cost function and estimated as:

$$SRAC = \frac{C_V}{y} = \frac{\exp(\ln C_V)}{y}$$

$$SRMC = \frac{\partial C_V}{\partial y} = \frac{\partial \ln C_V}{\partial \ln y} \frac{C_V}{y} = AC\left[\alpha_y + \gamma_{yy} \ln y + \sum_{i=1}^{J} \gamma_{iy} \ln p_i + \gamma_{ky} \ln k + \gamma_{yN} N \right] \qquad 12)$$

These measures all take one factor of production as fixed (capital). Long-run measures of cost elasticity yield estimates of economies of size when network size is included in the cost function. This measure is defined in similar fashion to the short-run estimate of economies of size and can be derived from the long-run cost function, along with estimates of marginal and average long-run cost.

4. DATA AND VARIABLES

In order to estimate the cost functions and associated share equations described previously, a panel data set was assembled using publicly-available financial and operating data on U.S. transit firms contained in the National Transit Database (NTD), which is maintained by the Federal Transit Administration (FTA). The data set contains observations on 23 firms over 8 years (1996 to 2003) for a total of 184 observations. The group of firms included in the sample are similar to previous work by Karlaftis and McCarthy (2002). Using a large sample of firms from 1986 to 1994, they used hierarchical clustering, a data reduction technique, in order to identify six groups of transit systems with similar operating characteristics. The firms chosen for the present study are those falling into the second-largest size group, including all but the five or six largest bus systems in the U.S.[7]

The use of panel data was considered for practical reasons. Most notably, the use of a limited number of relatively homogeneous firms ensured that the estimates of returns to scale and production structure would not be distorted by the effects of a large,

[7] One area of concern in conducting this analysis is whether there any changes in the composition of the size group over the decade or so since data was collected for the Karlaftis-McCarthy study. Some systems have undoubtedly grown over this period, while others have split up into smaller operations. This point will be returned to later in the paper.

heterogeneous sample, as has been observed with previous cross-sectional studies. Additionally, the use of a panel data set allows for more efficient parameter estimates, owing to a larger sample size. A related benefit is that a larger sample size reduces the effects of collinearity, which has been considered a problem in the estimation of translog cost functions (Williams 1979; De Borger 1984).

A summary of the data, including system operating characteristics and factor prices, are provided in Table 2.

Table 2: Summary Data for Transit Firms

Operating Characteristics		
Variable	Mean	S.D.
Operating cost ('000s $)	137,116	45,428
Number of vehicles	730	244
Gallons of fuel ('000s)	6,998	2,333
Vehicle miles ('000s)	26,304	8,218
Passengers ('000s)	62,037	20,025
Factor Input Variables		
Variable	Mean	S.D.
Price of labor ($)	26.90	4.91
Price of fuel ($)	1.23	0.15
Price of materials ($)	39,250	15,797
Share of labor ($)	0.76	0.07
Share of fuel ($)	0.04	0.01
Share of materials ($)	0.20	0.07

Developing factor prices for the input factors to be included in the cost function proved somewhat difficult. The short-run cost function employed three factors (labor, materials, and fuel), while a measure of capital input was included for a long-run specification. Following previous studies, a measure of labor price was calculated from the existing data set by dividing all labor costs (operators' wages, other salaries and wages, and fringe benefits) by the number of employee work hours reported for each firm. A factor price for fuel was calculated using data from the U.S. Department of Energy (DOE) on number 2 distillate average diesel fuel prices for sales to end users by state. For states for which this data was not available, historical weekly on-highway diesel (no. 2) prices were substituted. In each case, all applicable state and federal fuel taxes were applied to arrive at an end user price. The materials variable was defined as a composite variable to capture all of the operating costs not attributable to labor or fuel. A factor price for materials was composed by dividing total materials cost by the number of vehicles operated by each firm. Lastly, a measure of annualized capital cost was calculated using the following formula, following FTA specifications:

$$K_i = \sum_k A_{ki} N_{ki} V_{0ki} \qquad 13)$$

where:
K_i = cost of capital in year i
N_{ki} = fleet size of vehicle type k in year i
V_{0ki} = price of a new vehicle of type k in year i
A_{ki} = an annualization factor for vehicle type k in year i, with
$A = r_i(1+r_i)^n / (1+r_i)^n - 1$
and where:
n = service life of a vehicle (7 years for small buses, 15 years for all others)
r_i = interest rate on high-yield municipal bonds in year i

The interest rate on high-yield municipal bonds was collected from the U.S. Statistical Abstract. New vehicle costs were based on year 2000 estimates of vehicle purchase costs listed in *Metro* magazine. Unfortunately, price data were unavailable for other years, so an assumption was made that the price of new vehicles was relatively constant in real terms over the analysis period. All price and cost data were deflated to 1996 levels using the GDP deflator.

5. RESULTS

The results for the estimation of cost functions using Cobb-Douglas and translog functional forms are presented in this section. Results for the Cobb-Douglas estimation

Table 3: Variable Labels and Definitions

Variable	Description
LN C_V	LN of variable cost (C_V)
LN C_T	LN of total cost
LN Pl	LN of price of labor
LN Pm	LN of price of materials
LN Pf	LN of price of fuel
LN k	LN of fixed capital factor (number of buses)
LN Pk	LN of price of capital
LN VM	LN of annual vehicle miles
LN Pax	LN of passenger boardings
LN Pl-Pm	LN of labor-materials interaction
LN Pl-Pf	LN of labor-fuel interaction
LN Pm-Pf	LN of materials-fuel interaction
LN Pl-Pk	LN of labor-capital interaction
LN Pm-Pk	LN of materials-capital interaction
LN Pf-Pk	LN of fuel-capital interaction
LN Pl-k	LN of labor-fixed capital interaction
LN Pm-k	LN of materials-fixed capital factor interaction
LN Pf-k	LN of fuel-fixed capital factor interaction
LN Pl-VM	LN of labor-vehicle miles interaction
LN Pm-VM	LN of materials-vehicle miles interaction
LN Pk-VM	LN of vehicle miles-capital interaction
LN Pf-VM	LN of fuel-vehicle miles interaction
LN k-VM	LN of vehicle miles-fixed capital factor interaction
LN Pl-pax	LN of labor-passengers interaction
LN Pm-pax	LN of materials-passengers interaction
LN Pf-pax	LN of fuel-passengers interaction
LN Pk-pax	LN of capital-passengers interaction
LN k-pax	LN of fixed capital factor-passengers interaction
LN Pl-N	LN of labor-network size interaction
LN Pm-N	LN of materials-network size interaction
LN Pf-N	LN of fuel-network size interaction
LN Pk-N	LN of capital-network size interaction
LN k-N	LN of fixed capital factor-network size interaction
LN VM-N	LN of vehicle miles-network size interaction
LN pax-N	LN of passengers-network size interaction
1/2 Pl-Pl	1/2 * LN of price of labor squared
1/2 Pm-Pm	1/2 * LN of price of materials squared
1/2 Pf-Pf	1/2 * LN of price of fuel squared
1/2 k-k	1/2 * LN of fixed capital factor squared
1/2 Pk-Pk	1/2 * LN of price of capital squared
1/2 VM-VM	1/2 * LN of vehicle-miles squared
1/2 pax-pax	1/2 * LN of passengers squared
1/2 N-N	1/2 * LN of network size squared
LN N	LN of network size
LN TTI	LN of Travel Time Index
LN P/B	LN of peak-to-base ratio
Time Trend	Time trend variable

are presented first, since they provide results for both the short-run and long-run specification. Then, attention is turned to the translog cost function, which allows for examination of a broader range of outputs, including substitution elasticities. Table 3 lists the variables used in the analysis, along with a short description of their meaning.

5.1 Cobb-Douglas Cost Functions

The results of the estimated short-run and long-run Cobb-Douglas cost functions are presented in Table 4.

The models labeled Model 1 and Model 2 represent short-run cost functions with the level of capital fixed. Model 1includes vehicle miles as a measure of output, while Model 2 designates passenger boardings as the output measure. Both of the short-run cost functions indicate that cost increases less than proportionately with output. The

Table 4: Short and Long-run Cobb-Douglas Cost Function Estimates

	Model 1[a]		Model 2[b]		Model 3[c]		Model 4[d]	
Variable	Coefficient	SE	Coefficient	SE	Coefficient	SE	Coefficient	SE
LN Pl	0.72 (***)	0.06	0.45 (***)	0.07	0.78 (***)	0.06	0.47 (***)	0.10
LN Pm	0.26 (***)	0.03	0.28 (***)	0.03	0.20 (***)	0.03	0.19 (***)	0.04
LN Pf	0.04	0.09	0.10	0.10	0.10	0.12	0.17	0.17
LN k	0.46 (***)	0.07	0.79 (***)	0.06				
LN VM	0.78 (***)	0.08			1.14 (***)	0.05		
LN Route Miles	-0.21 (***)	0.03	0.03	0.03	-0.20 (***)	0.03	0.35 (***)	0.03
LN TTI	-0.01	0.10	-0.20 (*)	0.12	-0.01	0.11	-0.37 (**)	0.16
LN P/B	-0.15 (***)	0.05	-0.24 (***)	0.06	0.01	0.05	0.02	0.08
Time Trend	0.00	0.00	0.01	0.01	-0.01	0.01	0.00	0.01
LN Pk					-0.14	0.28	0.15	0.40
LN Pax			0.26 (***)	0.04			0.54 (***)	0.05
Constant	-1.24	1.10	4.03 (***)	0.87	-4.09 (***)	0.93	2.01 (*)	1.14
Adjusted R^2	0.88		0.86		0.86		0.71	
D-W Statistic	2.14		2.46		2.28		2.57	
N = 184								

[a] Short-run, variable cost function with vehicle miles as output
[b] Short-run, variable cost function with passenger boardings as output
[c] Long-run, total cost function with vehicle miles as output
[d] Long-run, total cost function with passenger boardings as output
Notes: Pi = Price of input i, f = fuel, k = fixed capital factor (rolling stock), l = labor, m = materials, VM = vehicle miles, pax = passenger boardings, N = Network size

model with vehicle miles specified as output (a "produced" measures) exhibits moderately increasing returns to size, while the model with a measure of consumed output (passengers) appears to exhibit rather strong returns to traffic density. Costs appear to increase by only about 0.26 percent with every one percent increase in passenger boardings.

Models 3 and 4 include capital as a factor of production in order to estimate long-run returns to scale. Model 3, which again uses vehicle miles as an output measure, shows some indications of declining returns to scale, as the output elasticity measure is significantly greater than one. Model 4, the "consumed" measure of output, again shows fairly strong evidence of increasing returns, though not as strong as the short-run cost function estimated with passengers as output.

All four estimated cost functions have a reasonably good statistical fit, though Model 4 is slightly lower (0.71). Models 1 and 3, which include vehicle-miles as an output measure, both estimate the share of labor as an input to be above 70 percent. These estimates accord with the findings of previous studies, most of which have found the labor share to be in the range of roughly 60 to 80 percent of costs. Fuel is a relatively small input, accounting for 10 percent or less of costs[8]. The input share for capital is estimated at about 15 percent in model 4, which seems reasonable for U.S. bus systems[9]. The time trend variable does not appear significant, indicating no significant technological change over the study period. The market-related variables do not seem to

[8] It might be interesting to reestimate these models with more recent data, given the subsequent rise in fuel costs since 2003. Given the relatively price-inelastic demand for fuel, the input share for fuel may have increased.

[9] There does not seem to be a good explanation for why the capital price coefficient in Model 3 is negative and insignificant.

have a consistent effect on costs. It is possible that the travel time index variable is confounded by other production factors in the model.

5.2 The Translog Cost Function

Estimates of the translog variable cost function are given in Table 5.

Table 5: Translog Variable Cost Function Estimates

	Model 1[a]		Model 2[b]		Model 3[c]	
Variable	Coefficient	SE	Coefficient	SE	Coefficient	SE
LN Pl	0.81	1.32	0.69	1.330	0.90	0.92
LN Pm	1.69	1.21	-0.64	1.11	0.10	0.92
LN Pf	-1.50	1.27	0.94	1.24	0.00	0.00
LN k	5.37 (***)	1.52	4.12 (***)	1.49	8.62 (***)	2.41
LN VM	2.60	1.60			-11.55 (***)	3.88
LN Pax			0.77	1.34		
LN Pl-Pm	-0.65 (***)	0.19	-0.06	0.23	-0.76 (***)	0.10
LN Pl-Pf	0.65 (***)	0.23	0.17	0.26	0.90 (***)	0.33
LN Pm-Pf	0.00	0.23	-0.12	0.25	0.00	0.12
LN Pl-k	-0.22	0.18	-0.66 (***)	0.17	-0.06	0.12
LN Pm-k	0.01	0.12	-0.08	0.09	0.24 (***)	0.08
LN Pf-k	-0.12	0.22	0.05	0.23	-0.30 (***)	0.11
LN Pl-VM	-0.12	0.15			0.29 (***)	0.11
LN Pm-VM	-0.23 (*)	0.12			-0.33 (***)	0.09
LN Pf-VM	0.35 (**)	0.15			0.04	0.07
LN k-VM	-0.04	0.21			-0.60 (***)	0.23
LN Pl-pax			0.11	0.11		
LN Pm-pax			-0.12 (**)	0.06		
LN Pf-pax			0.01	0.12		
LN k-pax			0.20 (*)	0.12		
LN Pl-N	0.32 (***)	0.12	0.19 (*)	0.11	-0.39 (***)	0.06
LN Pm-N	0.05	0.07	0.02	0.06	0.11 (***)	0.04
LN Pf-N	-0.37 (***)	0.13	-0.21	0.13	0.28 (***)	0.07
LN Pk-N						
LN k-N	-0.01	0.02	-0.01	0.02		
LN VM-N	0.13	0.08			0.15 (*)	0.09
LN pax-N			0.09 (*)	0.05		
1/2 Pl-Pl	0.84 (***)	0.23	0.17	0.26	-0.24	0.17
1/2 Pm-Pm	0.31 (***)	0.07	0.35 (***)	0.06	0.44 (***)	0.04
1/2 Pf-Pf	-0.41	0.32	0.13	0.37	-0.33	0.26
1/2 k-k	-0.43	0.34	-0.62 (***)	0.19	0.27	0.25
1/2 VM-VM	-0.09	0.16			1.00 (***)	0.32
1/2 pax-pax			-0.10	0.10		
1/2 N-N	0.00	0.05	0.02	0.03	-0.08 (**)	0.04
LN N	-2.09 (**)	1.02	-1.68	1.04	-3.36 (***)	1.21
LN TTI	0.07	0.07	0.01	0.08	-0.05	0.04
LN P/B	-0.04	0.03	-0.08 (***)	0.03	0.02	0.02
Time Trend	0.00	0.01	0.01	0.01	0.02 (**)	0.01
Constant	-28.76 (**)	12.58	-1.84	13.00	91.72 (***)	24.39

Adjusted R^2	0.92	0.90	0.67
N = 184			

[a] Short-run, variable cost function with vehicle miles as output
[b] Short-run, variable cost function with passenger boardings as output
[c] Same specification as model 1 with additional constraint that $0 < a_i < 1$
Notes: Pi = Price of input i, f = fuel, k = fixed capital factor (rolling stock), l = labor, m = materials, VM = vehicle miles, pax = passenger boardings, N = Network size

Again, Models and 1 and 2 represent cost functions with vehicle miles and passenger boardings specified as outputs, respectively[10]. These two models were estimated with the restrictions specified in equation (9), including linear homogeneity in factor prices, but without specified restrictions on the range of values for the input price coefficients. Thus, the values for these coefficients fail to fall within theoretically consistent ranges, and it is difficult to interpret their meaning. Model 3 includes vehicle miles as an output measure, and contains additional restrictions on the input price coefficients, forcing them to take on values between 0 and 1. These additional restrictions appear to significantly reduce the model fit.

Another interesting feature of the Model 3 estimates is that the restrictions on the input price coefficients seem to have forced the value of the fuel price coefficient to zero. Likewise, the labor share increased to 90 percent, larger than would be expected in nearly any transit firm.

All three of the translog cost functions in Table 5 share the characteristic of having a large number of variables with large standard errors, causing many individual variables to be marginally statistically significant or insignificant. Despite the large standard errors, the models retain a fairly good statistical fit. As noted earlier, this is

[10] While only short-run variable cost functions are specified here, estimates of long-run translog total cost functions were run, and are reported in Appendix A. Also, the translog cost function estimates reported in this section do not include the associated input share equation estimates. The model output for these equations is provided in Appendix B.

probably evidence of collinearity in the translog cost function. The fundamental structure of the model, including input factors with quadratic terms and cross-products, makes the presence of collinearity likely.

5.3 Factor Demand and Substitution

The output coefficients from Models 1 and 2 can be used to provide some estimates of the returns to size and capacity utilization[11]. Table 6 provides estimates of returns to size and capacity utilization for the translog cost function, along with the Cobb-Douglas cost function estimates for comparison.

Table 6: Returns to Scale Estimates for Translog and Cobb-Douglas Cost Functions

Short-run				
Model	RTCU[a]	RTSZ[b]	SRAC[c]	SRMC[d]
Cobb-Douglas Vehicle Miles	0.43	0.22	5.21	4.05
Cobb-Douglas Passengers	0.72	0.74	2.21	0.57
Translog Vehicle Miles	1.01	0.73	5.21	1.39
Translog Passengers	0.91	0.82	2.21	0.40
Long-run				
Model		RTSZ	LRAC	LRMC
Cobb-Douglas Vehicle Miles		-0.14	5.93	6.76
Cobb-Douglas Passengers		0.46	2.51	1.35

[a] Returns to capacity utilization
[b] Returns to size
[c] short-run average cost
[d] short-run marginal cost

Positive values for returns to size and capacity utilization provide evidence of increasing returns to scale. While the short-run Cobb-Douglas cost functions provided evidence of modest returns to scale, the estimates produced from the translog cost function indicate even stronger returns to scale, regardless of output. For both of the translog cost functions, marginal cost is significantly below average cost, indicating returns to both size and density.

[11] These estimates are still valid. Despite the large standard errors, these values remain consistent estimates of the true parameters. The value from Model 3 was excluded from the analysis due to its unusually large (and improbable) magnitude.

In addition to estimates of returns to scale, the translog cost function can provide estimates of factor price elasticities and substitution elasticities. The substitution elasticities can be calculated for the translog cost function using the Allen-Uzawa partial elasticities of substitution (Berndt 1991). For the translog specification, these are:

$$\sigma_{ij} = \frac{\gamma_{ij} + s_i s_j}{s_i s_j}, \quad i,j = 1,\ldots,J, \text{ for } i \neq j,$$

$$\sigma_{ii} = \frac{\gamma_{ii} + s_i^2 - s_i}{s_i^2}, \quad i = 1,\ldots J \qquad 14)$$

and the own price elasticity of demand is given by $(e_{ii}) = \sigma_{ii} s_I$, where s_I is the share of input *i* in costs. Estimates of the demand and substitution elasticities are shown in Table 7. The top half of Table 7 presents the estimates for Model 1, in which input price coefficients were not constrained to take values between zero and one, but all other restrictions were imposed to ensure linear homogeneity.

Table 7: Own-Price Elasticities and Elasticity of Substitution Estimates from Translog Cost Function

Partially-Constrained Estimates, Vehicle-Miles Output					
Own-Price Elasticities			Elasticities of Substitution		
e_{ll}	e_{mm}	e_{ff}	σ_{lm}	σ_{lf}	σ_{mf}
-0.08	-0.06	0.29	0.46	0.52	0.01
Fully-Constrained Estimates, Vehicle-Miles Output					
Own-Price Elasticities			Elasticities of Substitution		
e_{ll}	e_{mm}	e_{ff}	σ_{lm}	σ_{lf}	σ_{mf}
-0.41	N/A	0.51	N/A	-7.44	N/A

Note: l = labor, m = materials, f = fuel

According to the estimates, the demand for both labor and fuel is *highly* inelastic (both below 0.10). The demand elasticity for fuel had an unexpected positive sign. The estimated substitution elasticities for labor-materials and labor-fuel indicate that they are moderately substitutable, while materials and fuel are much less so.

The bottom half of Table 7 represents estimated own-price elasticities and substitution elasticities drawn from Model 3, which included additional constraints on the input price coefficients. As noted before, these additional constraints had the effect of inflating the labor input share and forcing the fuel share to zero. The zero input share for fuel meant that some of the elasticity estimates were unobtainable[12]. The own-price elasticity for labor demand is larger than the estimate produced by Model 1, but is still within a fairly demand-inelastic range. Also, the fuel share once again has an unusual positive sign. Two of the substitution elasticities could not be reasonably estimated due to the rather extreme values taken by the input price coefficients; the one that could be estimated (labor-fuel) was very large and negative. While labor and fuel are indeed complementary in production, a value less than -7 is unlikely to be observed.

6. APPLICATIONS OF THE COST FUNCTION

6.1 Costs and Pricing

We can use the information provided by the estimated cost functions to infer whether current pricing relates to the cost of providing transit service and how it might be changed. The most recent data (2003) for the Twin Cities metropolitan area indicates that the short-run average cost of a passenger trip was about $3.17. Both of the short-run cost functions using passengers as an output indicated evidence of strong economies of traffic density, thus marginal cost would be well below average cost. The Cobb-Douglas cost function predicts a marginal cost of $0.81 per trip, which is slightly below the average fare per boarding of around $0.90. The translog cost function, which allows returns to scale to vary with output levels, predicts a marginal cost of $1.23. Is current pricing efficient?

[12] It also likely had an influence on the rather large labor-fuel substitution elasticity.

There is good reason to believe not. First, current prices are set below even marginal cost, according to the translog estimates (though not the Cobb-Douglas). This is under an assumption of sharply increasing returns to scale, which is unlikely to be found in most bus systems. Karlaftis and McCarthy's (2002) previous study of the same group of transit systems studied in this paper found roughly constant returns to capacity utilization. This is a reasonable result, given that bus systems usually do not require large investments in fixed capital. Second, to the degree that increasing returns to density exist it is probably a reflection of existing overcapitalization of bus fleets. Viton's (1981) study estimated the degree of overcapitalization in bus fleets by interpreting the fixed production factor coefficient in a short-run, variable cost function. If the coefficient is positive, then adding vehicles to the fleet does not decrease costs, but rather contributes to overcapitalization. Looking at the estimates for Table 5, the fixed capital coefficient is positive and quite large, indicating a significant degree of overcapitalization. This is especially a problem for the Twin Cities, which has a high degree of peaking in its operations. Thus, the low marginal cost estimates could possibly reflect a large amount of excess capacity, as has been noted elsewhere (Walters 1982; Winston and Shirley 1998). McCarthy (2001) points out that the proper response to such a situation is to allow the fleet size to decline via attrition, rather than immediately dumping a large amount of rolling stock.

7. CONCLUSIONS

This paper has examined the cost structure of urban transit firms in the U.S. using two of the more widely accepted functional forms for cost functions: the Cobb-Douglas and the transcendental logarithmic (translog). Using a set of panel data covering 23 medium-large firms over an eight year period, the two functional forms were compared and used to produce estimates of returns to size, density, and scale by estimating both short and long-run cost functions.

While short-run cost functions were estimated using both functional forms, their estimates of returns to size and density varied based on the type of output measure employed. Both functional forms produced evidence of increasing returns to size, though the translog specification also produced estimates of rather large returns to traffic density. It is difficult to determine the exact source of increasing returns to density, though a hypothesis regarding significant amounts of excess capacity was offered, with special reference to the Twin Cities. A limited analysis of long-run costs using the Cobb-Douglas functional form produced evidence of increasing returns to scale with respect to passengers, but declining returns with respect to vehicle miles.

In characterizing the Cobb-Douglas and translog cost functions, it can be said that the Cobb-Douglas form provides a simpler model structure, is easier to estimate, and is less likely to violate the classical regression assumptions. It may be particularly useful in cases where the analyst must work with limited data. The translog cost function has the advantage of being more general, flexible, and theoretically appealing. It allows for estimates of returns to scale and production structure that require few *a priori* restrictions and can vary with output levels. However, the translog specification is more demanding in terms of estimation, since the cost function can entail dozens of parameter, depending on the number of inputs and outputs in the model. Also, the large number of related terms in the model can yield problems with collinearity, as was shown here.

In estimating the fully-constrained translog cost function, it was noted that the fit of the cost function was not as good. Several parameters were forced to unreasonable values and the overall model fit declined somewhat. This lead to problems with further interpretation of the output, particularly as applied to understanding production technology. While it is possible that the model simply did not fit the data well, it is equally possible that the data themselves were faulty enough to make the model unworkable. It was noted that some of the factor price variables had to be constructed from the existing data set, while good proxies for other factor prices were not available (as in the case of estimates of rolling stock costs).

There are a number of ways to improve upon this study in order to make the results more relevant. One would be to estimate a multi-output cost function for the firms included in this study. A number of the systems under study also operate light rail, commuter rail, heavy rail, or demand-responsive systems. Estimating cost functions for these systems could shed light on the issue of joint costs and economies of scope in transit provision (Viton 1992). Another would be estimate cost functions that include variables describing output characteristics such as trip length, in order to determine the effects of different types of operations on costs. Perhaps the most useful improvement would be to compare the results obtained in this study to the results from alternative approaches to measuring cost and efficiency. New methodological techniques have been developed in recent years to improve the ability of researchers to understand the nature of productive efficiency in firms. Econometric techniques such as stochastic production frontier analysis and mathematical programming approaches such as data envelopment analysis are gaining widespread acceptance and beginning to supplant more traditional cost function analyses in understanding firm cost and production capabilities.

REFERENCES

Berechman, Joseph. 1993. Public Transit Economics and Deregulation Policy. Amsterdam: North-Holland.

Berechman, Joseph and Genevieve Giuliano. 1984. "Analysis of the Cost Structure of an Urban Bus Transit Property." *Transportation Research,* 18B: 273-287.

_____. 1985. "Economies of Scale in Bus Transit: A Review of Concepts and Evidence." *Transportation* 12: 313-332.

Berndt, Ernst. 1991. The Practice of Econometrics: Classic and Contemporary. New York: Addison-Wesley.

Braeutigam, R.R. 1999. "Learning About Transport Costs." in Jose A. Gomez-Ibanez, William Tye, and Clifford Winston eds., Essays in Transportation Economics and Policy. Washington, D.C.: Brookings Institution Press.

Button, Kenneth J. 1993. Transport Economics, 2nd edition. Brookfield, VT: Elgar.

Button, Kenneth J. and K.J. O'Donnell. 1985. "An Examination of the Cost Structures Associated with Providing Urban Bus Services in Britain." *Scottish Journal of Political Economy* 32: 67-81.

Christensen, Laurits R., Dale W. Jorgenson, and Lawrence J. Lau. 1973. "Transcendental Logarithmic Production Frontiers." *Review of Economics and Statistics* 55(1): 28-45.

Colburn, Christopher and Wayne K. Talley. 1992. "A Firm-Specific Analysis of Economies of Size in the U.S. Urban Multiservice Transit Industry." *Transportation Research,* 26B(3): 195-206.

De Rus, Gines. 1990. "Public Transport Demand Elasticities in Spain." *Journal of Transport Economics and Policy* 24(2): 189-201.

De Borger, Bruno. 1984. "Cost and Productivity in Regional Bus Transportation: the Belgium Case Study." *Journal of Industrial Economics* 37(1): 35-54.

Johnston, J. 1956. "Scale, Costs, and Profitability in Road Passenger Transport." *Journal of Industrial Economics* 5(3): 207-222.

Karlaftis, Matthew G. and Patrick S. McCarthy. 2002. "Cost Structure of Public Transit Systems: A Panel Data Analysis." *Transportation Research, Part E* 38(1): 1-18.

Karlaftis, Matthew G., P.S. McCarthy, and K.C. Sinha. 1999a. "System Size and Cost Structure of Transit Industry." *ASCE Journal of Transportation Engineering* 125(3): 208-215.

_____. 1999b. "The Structure of Public Transit Costs in the Presence of Multiple Serial Correlation." *Journal of Transportation and Statistics* 2: 113-121.

Kumbhakar, Subal C. and Arunava Bhattacharyya. (1996) "Productivity Growth in Passenger-Bus Transportation: A Heteroskedastic Error Component Model with Unbalanced Panel Data." *Empirical Economics* 21(4): 557-73.

Lee, N. and I. Steedman. 1970. "Economies of Scale in Bus Transport I: Some British Municipal Results." *Journal of Transport Economics and Policy* 4(1): 15-28.

McCarthy, Patrick S. 2001. Transportation Economics: Theory and Practice: A Case Study Approach. Malden, MA: Blackwell Publishers.

Nerlove, Marc. (1963) "Returns to Scale in Electricity Supply." In Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld, edited by Carl F. Christ et al., pp. 167-198. Stanford, CA: Stanford University Press.

Obeng, Kofi. 1984. "The Economics of Bus Transit Operation." *Logistics and Transportation Review* 20(1): 45-65.

_____. 1985. "Bus Cost, Productivity, and Factor Substitution." *Journal of Transport Economics and Policy* 19(2): 183-203.

Pozdena, Randall and Leonard Merewitz. 1978. "Estimating Cost Functions for Rail Rapid Transit Properties." *Transportation Research* 12(2): 73-78.

Schrank, David and Tim Lomax. 2005. "The 2005 Urban Mobility Report." College Station, TX: Texas Transportation Institute.

Viton, Philip A. 1980. "On the Economics of Rapid-Transit Operations." *Transportation Research, Part A* 14A(4): 247-253.

_____. 1981. "A Translog Cost Function for Urban Bus Transit." *Journal of Industrial Economics* 29(3): 287-304.

_____. 1992. "Consolidations of Scale and Scope in Urban Transit." *Regional Science and Urban Economics* 22(1): 25-49.

Wabe, S. and O.B. Coles. 1975. "The Short and Long-run Costs of Bus Transport in Urban Areas." *Journal of Transport Economics and Policy* 9(2): 127-140.

Walters, Alan A. 1982. "Externalities in Urban Buses." *Journal of Urban Economics* 11(1): 60-72.

Williams, Martin. 1979. "Firm Size and Operating Costs in Urban Bus Transportation." *Journal of Industrial Economics* 28(2): 209-218.

Williams, M. and A. Dalal. 1981. "Estimation of the Elasticities of Factor Substitution in Urban Bus Transportation: A Cost Function Approach." *Journal of Regional Science* 21(2): 263-275.

Winston, Clifford and Chad Shirley. 1998. <u>Alternate Route: Toward Efficient Urban Transportation</u>. Washington, D.C.: Brookings Institution Press.

Zellner, A. 1962. "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias." *Journal of the American Statistical Association* 58(2): 348-368.

Appendix A: Long-Run Translog Cost Function Estimates

	Model 1[a]		Model 2[b]	
Variable	Coefficient	SE	Coefficient	SE
LN Pl	1.96	1.96	3.56	2.81
LN Pm	3.35 (**)	1.56	-2.76	2.12
LN Pf	-2.09	1.84	0.68	2.80
LN Pk	-2.22	2.12	-0.47	3.07
LN VM	4.17 (**)	1.96		
LN Pax			-7.95 (***)	2.28
LN Pl-Pm	-0.90 (***)	0.25	-0.12	0.46
LN Pl-Pf	0.75	0.76	1.21	1.21
LN Pm-Pf	-0.03	0.33	-0.45	0.52
LN Pl-Pk	1.48	1.16	-0.59	1.95
LN Pm-Pk	-0.59	0.51	0.79	0.81
LN Pf-Pk	-0.72	1.12	-0.85	1.73
LN Pl-VM	-0.38 (**)	0.16		
LN Pm-VM	-0.20 (*)	0.12		
LN Pk-VM	0.31	0.24		
LN Pf-VM	0.27	0.25		
LN Pl-pax			-0.28	0.18
LN Pm-pax			0.03	0.11
LN Pf-pax			-0.04	0.25
LN Pk-pax			0.29	0.24
LN Pl-N	0.35 (**)	0.16	-0.18	0.20
LN Pm-N	-0.02	0.10	-0.15 (*)	0.08
LN Pf-N	-0.39 (*)	0.20	-0.13	0.21
LN Pk-N	0.06	0.18	0.45 (**)	0.20
LN VM-N	-0.05	0.11		
LN pax-N			-0.01	0.09
1/2 Pl-Pl	1.34 (***)	0.40	0.44	0.66
1/2 Pm-Pm	0.23 (***)	0.08	0.40 (***)	0.12
1/2 Pf-Pf	0.00	0.64	0.52	0.92
1/2 k-k				
1/2 Pk-Pk	-0.77	1.79	-5.58 (**)	2.72
1/2 VM-VM	-0.07	0.16		
1/2 pax-pax			0.50 (***)	0.14
1/2 N-N	0.15 (**)	0.07	-0.14 (***)	0.05
LN N	0.36	1.31	3.67 (**)	1.87
LN TTI	0.05	0.10	-0.54 (***)	0.14
LN P/B	0.07 (*)	0.04	0.07	0.06
Time Trend	-0.01	0.01	0.00	0.01
Constant	-43.28 (**)	16.70	73.16 (***)	23.78
Adjusted R^2	0.92		0.80	
N = 184				

[a] Long-run, total cost function with vehicle-miles as output

[b] Long-run, total cost function with passenger boardings as output

Notes: Pi = Price of input i, f = fuel, k = capital (rolling stock), l = labor, m = materials, VM = vehicle miles, pax = passenger boardings, N = Network size

Appendix B: Input Factor Share Equation Estimates

Partially-Constrained Estimates with Vehicle Miles as Output				
	Labor Share		Fuel Share	
Variable	Coefficient	SE	Coefficient	SE
LN Pl	0.16 (***)	0.01	-0.04 (***)	0.01
LN Pm	-0.12 (***)	0.01	-0.01 (***)	0.00
LN Pf	-0.03	0.02	0.04 (***)	0.01
LN VM	0.03 (***)	0.01	0.00	0.00
LN N	-0.04 (***)	0.01	0.00	0.00
Constant	1.36 (***)	0.21	0.16 (***)	0.05
Adjusted R^2	0.77		0.55	
N = 184				
Partially-Constrained Estimates with Passengers as Output				
	Labor Share		Fuel Share	
Variable	Coefficient	SE	Coefficient	SE
LN Pl	0.16 (***)	0.02	-0.04 (***)	0.01
LN Pm	-0.13 (***)	0.01	-0.01 (***)	0.00
LN Pf	-0.02	0.02	0.04 (***)	0.01
LN Pax	0.02 (**)	0.01	0.00	0.00
LN N	-0.02 (**)	0.01	0.00	0.00
Constant	1.60 (***)	0.17	0.14 (***)	0.04
Adjusted R^2	0.76		0.55	
N = 184				
Fully-Constrained[a] Estimates with Vehicle Miles as Output				
	Labor Share		Fuel Share	
Variable	Coefficient	SE	Coefficient	SE
LN Pl			-0.03 (***)	0.00
LN Pm	-0.13 (***)	0.01	-0.01 (***)	0.00
LN Pf	0.11 (***)	0.02		
LN VM	0.05 (**)	0.02	-0.01 (***)	0.00
LN N	-0.06 (***)	0.01	0.00	0.00
Constant	1.07 (***)	0.28	0.32 (***)	0.04
Adjusted R^2	0.61		0.50	
N = 184				

[a] Includes constraint specifying that $0 < a_i < 1$

Notes: Pi = Price of input i, f = fuel, l = labor, m = materials, VM = vehicle miles, pax = passenger boardings, N = Network size